ODR n-tého řádu SPS

$$f = e^{\mu x}\,(P(x)\cos(\nu x) + Q(x)\sin(\nu x))$$

$y'' + 9y = 8\cos x \qquad y(0) = 0 \quad y'(0) = 1$

$= x_0$

- $y'' + 9y = 0 \qquad y_H = c_1 e^{0x}\cos(3x) + c_2 e^{0x}\sin(3x)$

  $\lambda^2 + 9 = 0 \qquad c_{1,2},\ x \in \mathbb{R}$

  $\lambda = 0 \pm 3i$

$$8\cos x = e^{\overset{\mu}{0}x}(\overset{P(x)}{8}\cos(\overset{\nu}{1}x) + \overset{Q(x)}{0}\sin(1x)) \qquad st(P) = st(Q) = 0$$

$$y_P = x^0 e^{0x}(A\cos(1x) + B\sin(1x))$$

$\mu + i\nu = 0 + i1$ je $0+i$ kořen? ne

$\neq\ 0 + 3i$ 1-násob. $\longrightarrow x^1$

$y_P = A\cos x + B\sin x \qquad y'' + 9y = 8\cos x$

$y_P' = -A\sin x + B\cos x$

$y_P'' = -A\cos x - B\sin x$

$$-A\cos x - B\sin x + 9A\cos x + 9B\sin x = 8\cos x\ +0\sin x$$

$\cos x$: $\quad -A + 9A = 8 \qquad A = 1$

$\sin x$: $\quad -B + 9B = 0 \qquad B = 0$

$y_P = 1\cos x + 0\sin x = \cos x$

Závěr: $\quad y = y_H + y_P$

$y = c_1\cos(3x) + c_2\sin(3x) + \cos x$

$x \in \mathbb{R} \qquad c_{1,2} \in \mathbb{R}$

$y(0)=0 \qquad y'(0)=1$

$$y' = c_1(-\sin(3x)\cdot 3) + c_2\cos(3x)\cdot 3 - \sin x$$

$0 = y(0) = c_1 + c_2\cdot 0 + 1 \qquad c_1+1=0 \qquad c_1=-1$

$1 = y'(0) = c_1\cdot 0 + c_2\cdot 3 - 0 \qquad 3c_2 = 1 \qquad c_2 = 1/3$

P.podají: $\quad y = -1\cos 3x + \underline{\underline{1/3 \sin 3x}} + \cos x$

$x \in \mathbb{R}$

$$y'' + y = x + \sin x$$

$y'' + y = 0 \qquad y_H$

$$\left.\begin{array}{ll} y'' + y = x & y_{P1} \\ y'' + y = \sin x & y_{P2} \end{array}\right\} \quad y = y_H + y_{P1} + y_{P2}$$

$$y'' - 2y' + y = \frac{e^x}{x} \qquad x \in (-\infty, 0) \quad x \in (0, \infty)$$

- $y'' - 2y' + y = 0 \qquad y_H = c_1 e^{1x} + c_2 x e^{1x}$

$$\lambda^2 - 2\lambda + 1 = 0$$

$$(\lambda - 1)^2 = 0$$

$\lambda_1 = 1$ 2-nás.

- $y_p = c_1(x) e^x + c_2(x) x e^x$

$$\to \quad y_p' = \underline{c_1' e^x} + c_1 e^x + \underline{c_2' x e^x} + c_2(e^x + x e^x)$$

Trik $\to \quad c_1' e^x + c_2' x e^x = 0$

$$y_p'' = c_1' e^x + c_1 e^x + c_2'(e^x + x e^x) + c_2(e^x + e^x + x e^x)$$

$$c_1' e^x + \cancel{c_1 e^x} + c_2'(x e^x + e^x) + c_2(\cancel{2e^x} + \cancel{x e^x}) - 2(\cancel{c_1 e^x} + c_2(\cancel{e^x} + \cancel{x e^x})) + \cancel{c_1 e^x} + c_2 \cancel{x} e^x = \frac{e^x}{x}$$

$$\to \quad c_1' e^x + c_2'(x e^x + e^x) = \frac{e^x}{x}$$

$$c_1' e^x + c_2' x e^x = 0 \qquad | \cdot (-1)$$

$$0 + 0 + c_2' e^x = \frac{e^x}{x} \qquad c_2' = \frac{1}{x} \qquad \underline{c_2 = \ln|x| + D_2}$$

$$c_1' e^x = -x e^x \cdot \frac{1}{x} \qquad c_1' = -1 \qquad \underline{c_1 = -x + D_1}$$

Dohromady: $y = (-x + D_1) e^x + (\ln|x| + D_2) x e^x$

$$x \in (-\infty, 0), \quad x \in (0, \infty)$$