Homogenní

$y' = f\left(\frac{y}{x}\right)$

$z = \frac{y}{x}$ $(\hookrightarrow$ sep. prom.$)$

PP $y(2) = 6$

Př. $2xyy' = 3y^2 - x^2$ $\quad x \in \mathbb{R}$

$: x^2$!

$x \in (-\infty, 0)$

$x \in (0, \infty)$

$2\frac{y}{x}y' = 3\left(\frac{y}{x}\right)^2 - 1$

$z = \frac{y}{x}$ $\quad y = x \cdot z$ $\quad y' = z + xz'$

$2z(z + xz') = 3z^2 - 1$

$2zxz' = z^2 - 1$ $\quad z \equiv 0$ ✗ $\quad 0 = -1$ ✗

$z' = \frac{z^2 - 1}{2zx}$ $\leftarrow g(y)$ $\quad x \in I_1 = (-\infty, 0)$, $I_2 = (0, \infty)$

$z \equiv 1$ $\quad z \equiv -1$ ✓

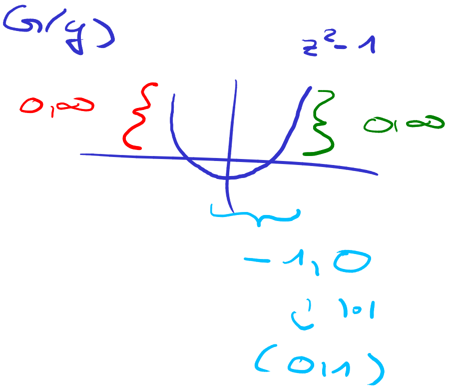
z: -1, 1

$J_1 = (-\infty, -1)$ $\quad J_2 = (-1, 1)$

$J_3 = (1, \infty)$

$$\int \frac{2z}{z^2 - 1}\,dz = \int \frac{1}{x}\,dx$$

$G(y) \rightarrow$ $\ln|z^2 - 1| = \ln|x| + k$



$z \in J_1$ $\quad G(J_1) = \mathbb{R}$

$z \in J_2$ $\quad G(J_2) = (-\infty, 0)$

$z \in J_3$ $\quad G(J_3) = \mathbb{R}$

$\ln|x| + k < 0$

$x \in (-\infty, 0)$: $\quad \ln|x| < -k$

$-x < e^{-k}$

$-e^{-k} < x$

$x \in (0, \infty)$: $\quad \ln|x| < -k$ $\quad x < e^{-k}$

$$\ln|z^2-1| = \ln|x| + t$$

$$|z^2-1| = e^t \cdot e^{\ln|x|}$$

- $z \in J_1$

$$z^2-1 = e^t|x|$$

$$z^2 = e^t|x|+1$$

$$z = -\sqrt{e^t|x|+1}$$

$z \in J_3$

$$z^2-1 = e^t|x|$$

$$\vdots$$

$$z = \sqrt{e^t|x|+1}$$

$\to$ $x \in (0,\infty)$, $x \in (-\infty,0)$

- $z \in J_2$

$$-z^2+1 = e^t|x|$$

$$1-e^t|x| = z^2$$

$$\pm\sqrt{1-e^t|x|} = z$$

$x \in (-e^{-t},0)$

$x \in (0,e^{-t})$

- $z = 1$ $\quad z = -1$

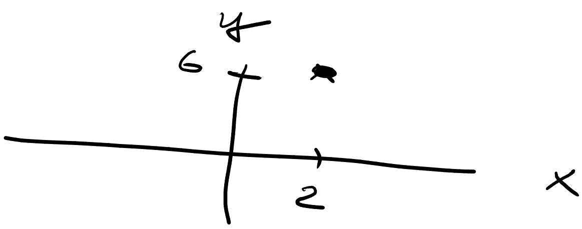
- $y = xz$

$$y = \pm x$$

$\to$ $y = \pm x\sqrt{1+|x|e^t}$ $\quad$ } $x \in (-\infty,0)$, $(0,\infty)$

$y = \pm x\sqrt{1-|x|e^t}$ $\quad$ } $x \in (-e^{-t},0)$, $x \in (0,e^{-t})$

$y(2) = 6$



$$6 = \pm 2\sqrt{1+2e^t}$$

$$3 = \sqrt{1+2e^t}$$

$$9 = 1+2e^t$$

$$4 = e^t$$

$$\boxed{\ln 4 = t}$$

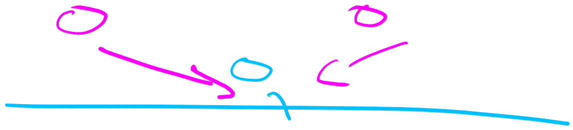
$$6 = 2\sqrt{1-2e^t}$$

$$9 = 1-2e^t$$

$$-4 = e^t \quad \times$$

Max. řešení:

$$y = \begin{cases} \pm x & \pm x\sqrt{1 - x e^{x}} \\ x\sqrt{1+4x} & x \in (0, \infty) \end{cases}$$



spoj ✓

+1

−1

$$\left(x\sqrt{1+4x}\right)' = \sqrt{1+4x} + \frac{x \cdot \frac{1}{2} \cdot 4}{\sqrt{1+4x}} \quad (*)$$

$$\lim_{x \to 0+} * = 1$$

$[0, 0]$, $y' = 1$

$$\pm\left(x\sqrt{1 - x e^{x}}\right)' = \pm\left(\sqrt{1 - x e^{x}} + \frac{x \cdot \frac{1}{2} \cdot (-e^{x})}{\sqrt{1 - x e^{x}}}\right)$$

$$\lim_{x \to 0-} = \pm 1)$$

$$2xy\,y' = 3y^2 - x^2$$

$$0 = 3 \cdot 0 - 0 \quad ✓$$

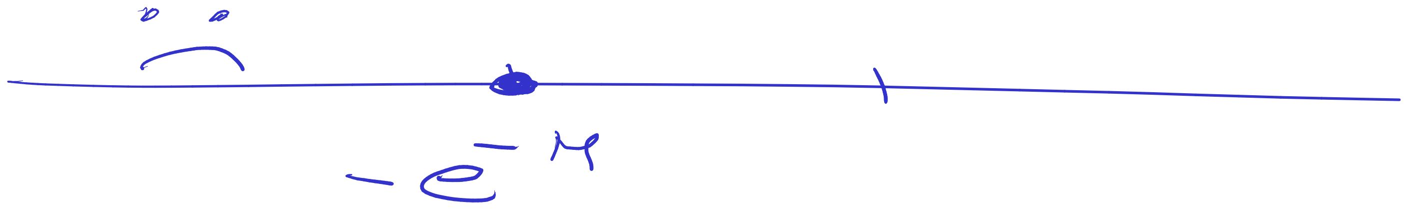
Závěr:

$$y = \begin{cases} x & x \le 0 \\ x\sqrt{1+4x} & x > 0 \end{cases} \qquad y = \begin{cases} x\sqrt{1 - x e^{x}} & x \le 0 \\ x\sqrt{1+4x} & x > 0 \end{cases}$$

méně než na $\mathbb{R}$

$$y = \begin{cases} x\sqrt{1 - (-x) e^{x}} & x \in (-e^{-x}, 0) \\ x\sqrt{1+4x} & x > 0 \end{cases}$$

$x\sqrt{1+|x|}\,e^{1}$



Lin dif. rce $\quad y' + p(x)\,y = q(x)$

P, q spoj. (a,b)

Př. $\quad y' - \frac{y}{x} = x \qquad p(x) = -\frac{1}{x} \qquad q(x) = x$

$x \in (-\infty,0) \qquad x \in (0,\infty)$

(1) $\quad y' - \frac{y}{x} = 0$

$y' = y \cdot \frac{1}{x} \qquad y \equiv 0$

$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$\ln|y| = \ln|x| + L$

$|y| = e^{\ln|x|} e^{L}$

$y = \pm |x| e^{L}$

$y = x \cdot M \qquad M \in \mathbb{R}$

(2) $\quad y = x \cdot M(x) \qquad y' - \frac{y}{x} = x$

$M(x) + xM'(x) - \frac{xM(x)}{x} = x$

$M'(x) = 1$

$\boxed{M(x) = x + D}$

(3) $\quad y = x(x+D) = x^2 + Dx \qquad x \in (-\infty,0)$

$x \in (0,\infty)$:)