ODR $\quad y' = g(y)h(x)$

- $y' = x^2 \qquad y = \frac{x^3}{3} + c$
- $y' = y \qquad y = k \cdot e^x$

$y(\underset{x_0}{0}) = \underset{y_0}{4} \qquad 4 = y(0) = ke^0$

$k = 4$

$y = 4e^x.$

Př. $\quad y' = -\frac{y}{x}$

$y' = y \cdot \frac{-1}{x}$

$g(y) = y \qquad h(x) = \frac{-1}{x}$

- $x \neq 0 \qquad x \in \underset{I_1}{(-\infty, 0)} \quad \underset{I_2}{(0, \infty)}$
- $y \equiv 0$ (y je konst. 0)

$0 = -\frac{0}{x}$ ✓ $\quad x \in I_1, \quad x \in I_2$

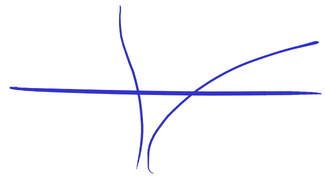
- $J_1 = (-\infty, 0) \quad J_2 = (0, \infty)$

$y:$

• $y' = y \cdot \frac{-1}{x}$ • Fix I_1, J_1

$$\frac{1}{y}\, y' = -\frac{1}{x}$$

$$\frac{dy}{dx}$$



$$\int \frac{1}{y}\, dy = \int -\frac{1}{x}\, dx$$

$$\ln|y| = -\ln|x| + k$$

$G(y) = \ln|y|$ $H(x) = -\ln|x|$

Fix x $I_1 = (-\infty, 0)$ $G(J_1) = \mathbb{R}$

y $J_1 = (-\infty, 0)$ $\forall x \in I_1 :\ -\ln|x| + k \in \mathbb{R}$ ✓

k

$$|y| = e^{+\ln|x|^{-1} + k}$$

$$|y| = |x|^{-1} \cdot e^{k}$$

$$y = \pm \frac{e^{k}}{|x|}$$

I_1 J_2

$G(J_2) = \mathbb{R}$

I_2 J_1

I_2 J_2

$\times$ ✓

Závěr: $y = \pm \frac{e^{k}}{|x|}$ $x \in (-\infty, 0)$

$x \in (0, \infty)$

$y = 0$ x — " —

Pf. $y' = \sqrt[3]{y^2} \cdot 1$

$g(y) = y^{2/3}$ $\quad h(x) = 1$

- $x \in \mathbb{R} = I$
- $y = 0$ ✓ $\quad x \in \mathbb{R}$
- $J_1 = (-\infty, 0) \quad J_2 = (0, \infty)$
-

$$\int \frac{1}{y^{2/3}}\, dy = \int 1 \, dx$$

$$\underbrace{3\sqrt[3]{y}}_{G(y)} = x + k$$

- $J_1 = (-\infty, 0)$ $\quad$ fix k:

$G(J_1) = (-\infty, 0)$ $\quad x + k < 0$

$x < -k$

$$\sqrt[3]{y} = \frac{x+k}{3}$$

$$y = \left(\frac{x+k}{3}\right)^3 \qquad x \in (-\infty, -k)$$

- $J_2 = (0, \infty)$ $\quad$ fix k

$G(J_2) = (0, \infty)$ $\quad x + k > 0$

$x > -k$

$$y = \left(\frac{x+k}{3}\right)^3$$

kde chceme řešení: $x \in \mathbb{R}$

(a) $y = 0 \qquad x \in \mathbb{R}$

(b)
$$y = \begin{cases} \left(\frac{x+k}{3}\right)^3 & x < -k \\ 0 & x = -k \\ 0 & x > -k \end{cases}$$

(c)
$$y = \begin{cases} 0 & x < -k \\ 0 & \\ \left(\frac{x+k}{3}\right)^3 & x > -k \end{cases}$$

(d)
$$y = \begin{cases} \left(\frac{x+k}{3}\right)^3 & x < -k \\ 0 & x = -k \\ 0 & -m > x > -k \\ 0 & x = -m \\ \left(\frac{x+m}{3}\right)^3 & \end{cases}$$

$$-k \leq -m$$
$$m \leq k$$

- ? nalepeno spoj. ? 1 nemusí vyjít

(b) $\lim\limits_{x \to -k-} y(x) \stackrel{?}{=} \lim\limits_{x \to -k+} y(x)$

$$\lim_{x \to -k-} \left(\frac{x+k}{3}\right)^3 = 0 \stackrel{?}{=} \lim_{x \to -k+} 0 = 0 \checkmark$$

? existuje dco v -2 ? // mělo by vyjít

$$y'_+ = 0$$

$$y'_- = \lim_{x \to -2^-} \left(\left(\frac{x+2}{3}\right)^3\right)' =$$

$$= \lim_{x \to -2^-} 3\left(\frac{x+2}{3}\right)^2 \cdot \frac{1}{3} = 0$$

$\Big\}$ dco v -2 $\exists$ a $= 0$.

Závěr: slepeno dobře.

ostatní případy analog.