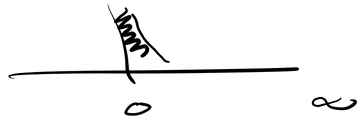
Neabsolutní konv. $\int$

$$\bullet \int_0^\infty \underbrace{\frac{\operatorname{arctan} x^2}{x^a} \sin(2x)}_{h(x)}\, dx \qquad a>0$$



problémy: ∞, 0

$$\int_0^\infty h = \int_0^{\pi/2} h + \int_{\pi/2}^\infty h$$

$$\circ \int_0^{\pi/2} \frac{\operatorname{arctan} x^2}{x^a} \sin(2x)\, dx$$ 1) nemění znaménko

LSK: $n(x) = \dfrac{x^2 \cdot x}{x^a} = x^{3-a}$; $\int_0^{\pi/2} x^{3-a}\, dx$

$$\lim_{x\to 0+} \frac{h}{n} = \lim_{x\to 0+} \frac{\frac{\operatorname{arctan} x^2}{x^a} \cdot \sin 2x}{\frac{x^2 \cdot x}{x^a}}$$

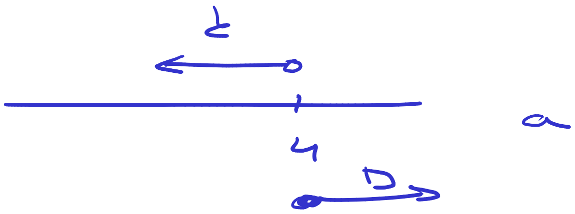
K $\Leftrightarrow 3-a > -1$ $\boxed{4 > a}$

$$= \lim_{x\to 0+} \frac{\operatorname{arctan} x^2}{x^2} \cdot \frac{\sin 2x}{2x} \cdot 2 = 2 \in (0,\infty)$$

$$\int_0^{\pi/2} h \text{ K} \Leftrightarrow \int_0^{\pi/2} n(x) \text{ K} \Leftrightarrow \boxed{a<4}$$

h, n spoj. na $(0, \pi/2]$

$n \geq 0$ na $(0, \pi/2]$



konv splývá s AK

$$\bullet \int_{\pi/2}^\infty \operatorname{arctan} x^2 \cdot \frac{1}{x^a} \cdot \sin(2x)$$

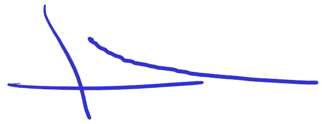
$$\int_{\pi/2}^{\infty} \underbrace{\frac{1}{x^a}}_{g} \underbrace{\sin(2x)}_{f} \, dx \qquad a > 0$$

D

f, g spoj $[\pi/2, \infty)$ ✓

$F = -\frac{1}{2}\cos(2x)$ om? ✓

g monot. $\frac{1}{x^a}$



✓

$\lim_{x \to \infty} \frac{1}{x^a} = 0$ ✓

$$\int_{\pi/2}^{\infty} \frac{1}{x^a} \sin(2x) \, dx \quad K$$

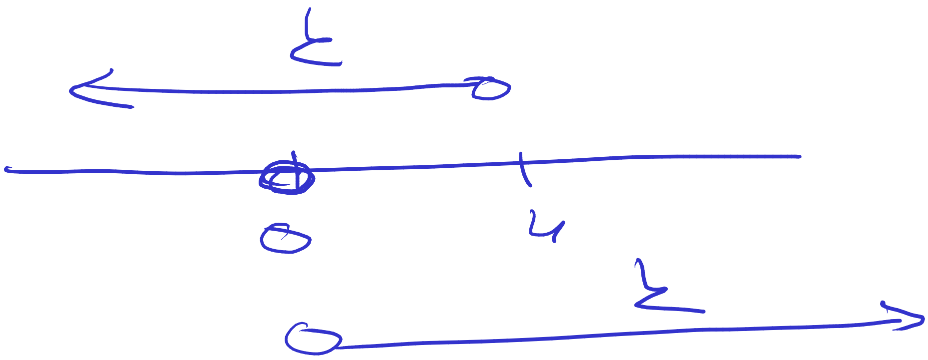
A

$$\int_{\pi/2}^{\infty} \underbrace{\operatorname{arctan}(x^2)}_{g} \underbrace{\frac{\sin(2x)}{x^a}}_{f} \, dx$$

g om. ✓ monot. ✓ $f \in N(a,b)$

$$\to \int_{\pi/2}^{\infty} \operatorname{arctan}(x^2) \frac{\sin(2x)}{x^a}$$

Závěr



$\int$ Ak pro $a \in (0,4)$

$$\int_{\pi/2}^{\infty} \frac{\arctan x^2}{x^a}\,|\sin(2x)|\, dx$$

Sk, podm.

$$\frac{\arctan x^2}{x^a}\,|\sin(2x)| \le \frac{\pi/2 \cdot 1}{x^a}$$

$\int \frac{\pi/2}{x^a}$ AK $\Leftrightarrow a > 1$

AK

?? 0 1 4

AK

$a \in (1,4)$ AK

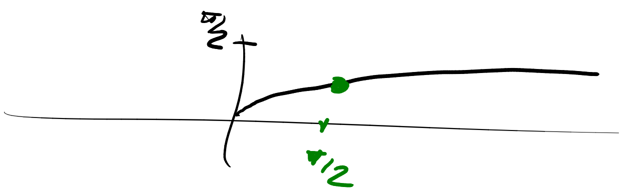
Co když $a \in (0,1)$

$$\int_{\pi/2}^{\infty} \frac{|\sin 2x|}{x^a}\, dx \quad D \quad \text{tabulka}$$

$y = 2x$

$dy = 2\,dx$

$$\int_{\pi}^{\infty} \frac{1}{2}\,\frac{|\sin y|}{\frac{1}{2^a}\, y^a}\, dy$$



$$\frac{\arctan x^2}{x^a}\,|\sin 2x| \ge \frac{|\sin 2x|}{x^a} \quad D$$

Sk (podm.) $\rightarrow$ $a \in (0,1]$ D $|\cdot|$

AK

0 $|\cdot|$ D 1 AK 4

k

Závěr: $\int$ AK pro $a \in (1,4)$

NAK $a \in (0,1]$

jinak D

$$\int_1^\infty \frac{\dots}{-2x}\,\underbrace{-2x\,e^{-x^2}}_{\text{má PF}}\,dx \qquad e^{-x^2}$$

$$\int_1^\infty \frac{e^x}{e^x}\,x^a e^{bx}\,dx \ \text{AK} \iff \text{AK}\ \int_e^\infty \frac{1}{y}(\ln y)^a\, y^b\,dy$$

$$y = e^x \qquad x = \ln y$$

$$dy = e^x\,dx$$

x	1	∞
y	e	∞

$$\varphi = e^x$$

$$\varphi : (1,\infty) \longrightarrow (e,\infty)$$