AK NI

? $\int_a^b |f(x)|\,dx = \infty$? $\int_0^\infty \left|\frac{\sin x}{x}\right| dx$

SK

$\sum |a_n| \le \sum b_n$ K $\quad$ K

$\sum b_n$ D $\le \sum a_n$ D

LSK

$\lim\limits_{n\to\infty} \frac{a_n}{b_n} = M \in (0,\infty) \qquad \sum a_n$? $\iff \sum b_n$ K

...

$\int_a^b f \le \int_a^b g \qquad \int_a^b g$ K $\Rightarrow \int_a^b f$

$\lim\limits_{x\to b-} \frac{f}{g} = M \in (0,\infty) \Rightarrow \int_a^b g$ K $\iff \int_a^b f$?

$\sum\limits_{n=1}^{\infty} n^\alpha$ K : $\alpha < -1 \qquad$ D $\alpha \ge -1$

$$\int_1^\infty x^\alpha\,dx = \begin{cases} \left[\frac{x^{\alpha+1}}{\alpha+1}\right]_1^\infty & \alpha \neq -1 \quad \begin{cases} 0 - \frac{1}{\alpha+1} \end{cases} \\ \left[\ln x\right]_1^\infty = \text{“}\ln\infty - \ln 1\text{”} = \infty & \alpha = -1 \end{cases}$$

$\boxed{\begin{matrix}\alpha+1<0 \\ \alpha<-1 \text{ K}\end{matrix}}$

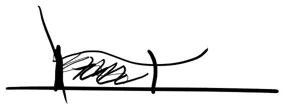
$\alpha \ge -1$ D

- $\displaystyle\int_0^\infty \underbrace{\frac{1}{1+x^2}}_{f} dx = [\arctan x]_0^\infty = \lim_{x\to\infty} \arctan x - \lim_{x\to 0+} \arctan x$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2} \qquad \int k$$

- f spoj. na om. uz. intervalu?

$f = \dfrac{1}{1+x^2}$. $(0,\infty)$ neom :(



- $\displaystyle\int_0^\infty f(x) = \underbrace{\int_0^1}_{k} + \underbrace{\int_1^\infty}_{k}$

$\displaystyle\int_0^1 \frac{1}{1+x^2} dx$ spoj. $[0,1]$ ✓ $\Rightarrow \displaystyle\int_0^1 f \; k$

$\displaystyle\int_1^\infty \underbrace{\frac{1}{1+x^2}}_{f}$ $\qquad \displaystyle\sum \frac{1}{1+n^2} \qquad \frac{1}{n^2}$

$g = \dfrac{1}{x^2} \qquad \displaystyle\int_1^\infty \frac{1}{x^2} \; k$

$$\lim_{x\to\infty} \frac{f}{g} = \lim \frac{\frac{1}{1+x^2}}{\frac{1}{x^2}} = 1 \in (0,\infty).$$

$\displaystyle\int_1^\infty f \; k \Leftrightarrow \int_1^\infty g$ $\qquad f, g$ spoj. $[1,\infty)$

$\dfrac{1}{x^2} \geq 0$ — " —

$\Rightarrow \displaystyle\int_1^\infty f \; k$:)

Zaklj: $\displaystyle\int_0^\infty f \; k$ ✓

$\displaystyle\int_1^\infty \frac{1}{1+x^2} \; k$ ✓ $\Leftarrow \displaystyle\int_1^\infty \frac{1}{x^2} \; k$

$\int k$

$0 \leq \frac{1}{1+x^2} \leq \frac{1}{x^2}$ ✓ na $[1, \infty)$

$\frac{1}{1+x^2}$ spoj. na $[1, \infty)$

$$\int_0^\infty \frac{1}{1+x^2}\,dx$$

$\frac{1}{x^2}$ spoj $[0, \infty)$? ✗ :(

$$\int_0^\infty \frac{1}{x^2} = \left[-\frac{1}{x}\right]_0^\infty = 0 - (-\infty) = \infty$$

$$\lim_{x \to \infty} \frac{f}{g} = 1$$

$$\int_0^{\pi/2} \underbrace{\frac{\sin x}{e^x - 1} \cdot \frac{1}{\arctan x}}_{f}\,dx$$

f spoj $(0, \frac{\pi}{2}]$

problémy v 0

$$g = \frac{x}{x \cdot x} = \frac{1}{x} \geq 0 \qquad \int_0^{\pi/2} \frac{1}{x}\,dx = \infty$$

f, g spoj na $(0, \frac{\pi}{2}]$

LSK

$$\lim_{x \to 0+} \frac{f}{g} = \lim_{x \to 0+} \frac{\frac{\sin x}{(e^x - 1)(\arctan x)}}{\frac{x}{x \cdot x}} =$$

$$= \lim_{x \to 0+} \frac{\sin x}{x} \cdot \frac{x}{e^x - 1} \cdot \frac{x}{\arctan x} = 1 \cdot 1 \cdot 1 = 1 \in (0, \infty)$$

$$\int_0^{\pi/2} g \ D \longrightarrow \int_0^{\pi/2} f \ D$$