Lin ODR $\qquad y' + p(x)y = q(x) \qquad$ p,q ciąg: (a, b)

- $y' - \frac{1}{x+2} y = x+2 \qquad x \neq -2 \qquad x \in (-\infty, -2)$, $x \in (-2, \infty)$

$$y' - \frac{1}{x+2} y = 0 \qquad \rightarrow \quad y \equiv 0$$

$$\frac{1}{y} y' = \frac{1}{x+2}$$

$$\int \frac{1}{y}\, dy = \int \frac{1}{x+2}\, dx$$

$$\ln|y| = \ln|x+2| + k \qquad k \in \mathbb{R}$$

$$|y| = |x+2| \cdot e^{k}$$

$$y = \pm M\, |x+2| \qquad M > 0$$

$$y = L(x+2) \qquad L \in \mathbb{R}$$

- $y_p = L(x)\,(x+2)$

$$y_p' = L'(x+2) + L \cdot 1$$

$$L'(x+2) + \cancel{L} - \frac{1}{x+2} \cancel{L(x+2)} = x+2$$

$$L' = 1 \qquad L = x + D$$

- Całka $\qquad y = (x+D)(x+2) \qquad x \in (-\infty, -2)$, $x \in (-2, +\infty)$

$$(x+2)y' - y = (x+2)^2 \qquad x \in \mathbb{R}$$

$$y' - \frac{y}{x+2} = (x+2) \qquad x \in (-\infty, -2) \cup (-2, \infty)$$

$$\Downarrow$$

$$y_1 = (x+D_1)(x+2) \qquad x < -2$$

$$y_2 = (x+D_2)(x+2) \qquad x > -2$$

$y_1 \quad y_2$

-2

Lepení

· spoj.

$$\lim_{x\to -2-} y_1 = \lim (x+D_1)(x+2) = 0 = \lim_{x\to -2+} y_2 = \lim (x+D_2)(x+2)$$

dů.

$$\lim_{x\to -2-} y_1' = D_1 + 2x + 2 = D_1 - 2 = \lim_{x\to -2+} y_2' = \lim D_2 + 2x + 2 = D_2 - 2$$

$$\boxed{D_1 = D_2}$$

Závěr $\quad y = (x+2)(x+D_1) \qquad D_1 \in \mathbb{R}$

kontrola ODR: $\quad x=-2 \qquad y(-2) = 0 \qquad y'(-2) = D_1 - 2$

$(x+2)y' - y = (x+2)^2 \quad$ v $x=-2$: $\quad 0\cdot(D_1-2) - 0 = 0$ ✓

$$y' - \frac{2}{x+2}\, y = 2(x+2)\sqrt{y} \qquad \begin{array}{l} x\in(-\infty,-2) \\ x\in(-2,\infty) \\ y \geq 0 \end{array}$$

$z = y^{1-\alpha} = y^{1-1/2} = \sqrt{y} \qquad y = z^2 \qquad y' = 2z\cdot z' \qquad y^{1/2} \qquad \alpha = 1/2$

$z \geq 0$

$2z\cdot z' - \frac{2}{x+2} z^2 = 2(x+2)z \qquad | : 2z \qquad z \equiv 0$ ✓

$z' - \frac{2}{x+2} = (x+2) \qquad z \neq 0$

⇓

$z = (x+2)(x+D) \qquad z \geq 0 \qquad 2 > D$: $x \in (-D,\infty)$, $(-\infty,-2)$

$y = (x+2)^2(x+D)^2 \qquad 2 < D \quad x\in(-2,\infty)$, $(-\infty,-D)$

$2 = D \quad x\in(-2,\infty)$, $(-\infty,-D)$

$y \equiv 0 \qquad x\in(-\infty,-2),\ x\in(-2,\infty)$

$y = (x+2)^2(x+D)^2$

lze slepit v $x=-D$?

ODR   n-tého řádu   s konst. koef

$x \in \mathbb{R}$

- $y''' - 6y'' + 9y' = 0$

$y = c_1 e^{0x} + c_2 e^{3x} + c_3 x e^{3x}$

$\lambda^3 - 6\lambda^2 + 9\lambda^1 = 0$ (λ číslo) $c_{1,2,3} \in \mathbb{R}$

$\lambda(\lambda - 3)^2 = 0$

$FSŘ = \{1, e^{3x}, xe^{3x}\}$

$\lambda_1 = 0$ $\lambda_{2,3} = 3$

1-nás   2-nás.

- $y^{(4)} + 8y'' + 16y = 0$

$\sqrt{(-1)(16)} = i4$

$\lambda^4 + 8\lambda^2 + 16 \cdot 1 = 0$

$\lambda_{1,2} = \frac{0 \pm \sqrt{0 - 16}}{2} = \frac{\pm 4i}{2}$

$(\lambda^2 + 4)^2 = 0$

$\lambda = 2i = 0 + 2i$ $\lambda = -2i = 0 - 2i$

2-nás   2 nás

$FSŘ = \left\{ \begin{array}{ll} e^{0x}\cos(2x), & e^{0x}\sin(2x) \\ x e^{0x}\cos(2x), & x e^{0x}\sin(2x) \end{array} \right\}$

$y = c_1 \cos 2x + c_2 \sin 2x + c_3 x \cos 2x + c_4 x \sin 2x$

$c_{1,2,3,4} \in \mathbb{R}$ $x \in \mathbb{R}$