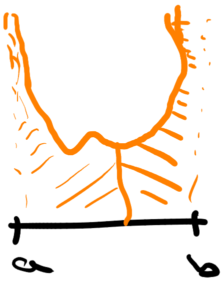
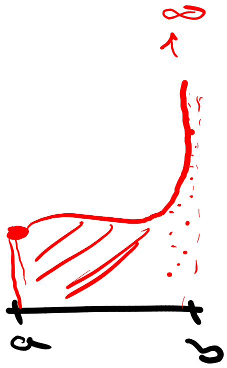
Ak ∼∫

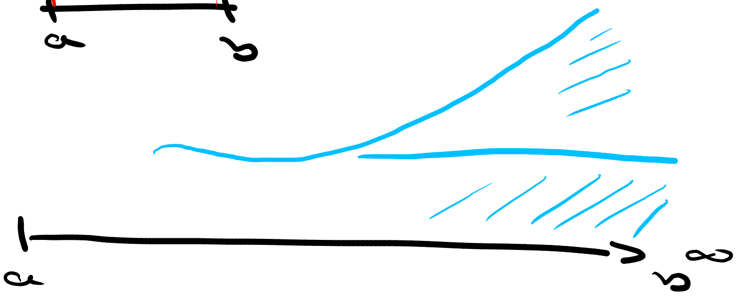
$\int_a^b |f(x)|$ k ?





spoj. ocu. uz.

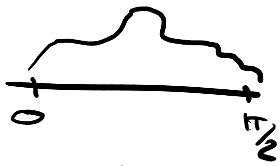
(L)SS $\frac{1}{x^2}$ D $\frac{1}{\sqrt{x}}$ K



(L)SS

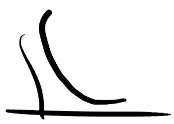
$\frac{1}{x^2}$ ⋮ K $\frac{1}{\sqrt{x}}$ ⋮ D

$$\int_0^{\pi/2} (\sin x)^p (\cos x)^q \, dx$$



$p = -3$

$$\frac{1}{\sin^3 x} \cdot \cos x$$



- $(0, \pi/4)$
- $(\frac{\pi}{4}, \pi/2)$

u 0:

$$\lim_{x \to 0} (\cos x)^q = 1^q$$

chová se jako:

$\sin x \approx x \qquad (\sin x)^p \approx x^p$

LSS

$g(x) = x^p \, 1^q$

$$\lim_{x \to 0+} \frac{(\sin x)^p (\cos x)^q}{x^p \cdot 1^q} = 1^p \cdot 1 \in (0, \infty) \qquad \int_0^{\pi/4} f \, d \iff \int_0^{\pi/4} g \, k$$

$\int_0^{\pi/4} x^p \, dx$ k $\iff$ $p > -1$

- f, g spoj. na $(0, \pi/4]$ ✓

  $g \geq 0$ — " — ✓

Záver: $\int_0^{\pi/4} f$ k $\iff$ $p > -1$

u $\frac{\pi}{2}$:

$$\lim_{x \to \frac{\pi}{2}-} (\sin x)^p = 1^p$$

$$(\cos x)^q \approx \left(\frac{\pi}{2} - x\right)^q$$

$\hookrightarrow$ $g(x) = \left(\frac{\pi}{2} - x\right)^q$

$$\lim_{x \to \frac{\pi}{2}-} \frac{(\sin x)^p (\cos x)^q}{\left(\frac{\pi}{2} - x\right)^q} = 1^p \cdot 1^q \in (0, \infty)$$

$$\lim_{x \to \frac{\pi}{2}-} \frac{\cos x}{\frac{\pi}{2} - x} \overset{L'H}{\underset{\frac{0}{0}}{=}} \lim_{x \to \frac{\pi}{2}-} \frac{-\sin x}{-1} = 1$$

$\int_{\pi/4}^{\pi/2} f$ k $\iff$ $\int_{\pi/4}^{\pi/2} g$ k

$\int_{\pi/4}^{\pi/2} \left(\frac{\pi}{2} - x\right)^q dx = \int_0^{\pi/4} y^q$ k $\iff$ $q > -1$

$y = \frac{\pi}{2} - x$

$dy = -1 \, dx$

x	$\pi/4$	$\pi/2$
y	$\pi/4$	0

Podm. f, g spoj $[\pi/4, \pi/2)$ ✓

$g \geq 0$ — " — ✓

Závěr: $\int_{\pi/4}^{\pi/2} f \downarrow \iff q > -1$

Dohromady $\int_{0}^{\pi/2} f \downarrow \iff (p > -1) \;\&\; (q > -1)$

$$\frac{x^p}{1+x^q}$$

$$(x^\alpha + x^3)$$