Určitý $\int$ – Newtonův $\int$

$$\int_a^b f\,dx = \lim_{x\to b-} F(x) - \lim_{x\to a+} F(x)$$

$$\int_0^1 \frac{1}{1+x^2}\,dx = \left[\operatorname{arctan}\right]_0^1 = \lim_{x\to 1-} \operatorname{arctan} x - \lim_{x\to 0+} \operatorname{arctan} x$$

$0 \to 1$

$$= \operatorname{arctan} 1 - \operatorname{arctan} 0 = \frac{\pi}{4}$$

$$\int_0^\infty \sin x\,dx = \left[-\cos x\right]_0^\infty = \lim_{x\to\infty} -\cos x - \lim_{x\to 0} -\cos x = !$$

$\int \nexists$

$$\int_0^\infty e^x\,dx = \left[e^x\right]_0^\infty = \lim_{x\to\infty} e^x - \lim_{x\to 0+} e^x = \infty - 1$$

$$= \infty \quad \text{Diverguje}$$

$$\int_{-\infty}^{\infty} x\,dx = \left[\frac{x^2}{2}\right]_{-\infty}^{\infty} = \infty - \infty$$

$\int \nexists$

$$\cancel{\int_{-3}^{3} \frac{1}{x}\,dx \overset{!}{=} \left[\ln|x|\right]_{-3}^{3} = \ln 3 - \ln|-3|}$$

$$= 0$$

$\frac{1}{x}$ nemá PF

na $(-3, 3)$

Per partes

$$\int_0^{\pi/2} \underset{u}{x}\, \underset{v'}{\sin x}\, dx = \Big[x(-\cos x)\Big]_0^{\pi/2} - \int_0^{\pi/2} -\cos x\, dx =$$

$u' = 1 \qquad v = -\cos x$

$$= \Big[x(-\cos x)\Big]_0^{\pi/2} + \Big[\sin x\Big]_0^{\pi/2}$$

$$= \frac{\pi}{2}\left(-\cos\frac{\pi}{2}\right) - 0\cdot(-\cos 0) + \sin\frac{\pi}{2} - \sin 0 =$$

$$= \underline{\underline{1}}\,.$$

Substituce

$$\int_0^2 \underbrace{\frac{1}{2\sqrt{x^2+1}}\cdot 2x}_{f(\varphi)\cdot\varphi'}\, dx = \int_1^5 \underbrace{\frac{1}{2\sqrt{y}}}_{f(y)}\, dy = \Big[\sqrt{y}\Big]_1^5$$

$$= \underline{\underline{\sqrt{5} - \sqrt{1}}}$$

$y = x^2 + 1 \leftarrow \varphi(x)$

$dy = \underbrace{2x}_{\varphi'(x)}\, dx$

x	0	2
$x^2+1 = y$	1	5

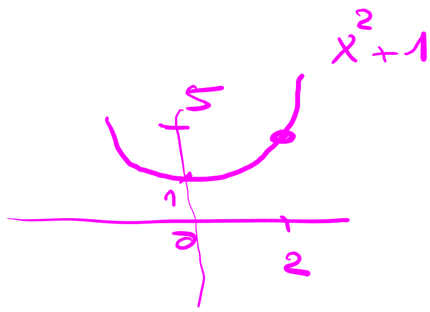
$(\alpha, \beta) = (0; 2)$

$(a, b) = (1, 5)$

$\frac{1}{2\sqrt{y}}$ spoj. $[1,5]$ ✓

$\varphi : [0,2] \xrightarrow{\text{do}} [1,5]$ ✓

$\varphi' = 2x$ spoj. $[0,2]$ ✓



$$\int_{-1/2}^{2} 2x+1\,\frac{2}{2}\, dx = \frac{1}{2}\int_0^5 y\, dy = \left[\frac{y^2}{2}\right]_0^5 = \frac{25}{2}$$

$y = 2x + 1$

$dy = 2\,dx$