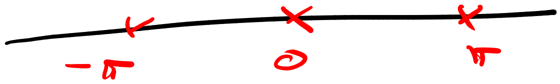
$\int \frac{\sin x}{1-\cos^2 x}\,dx$ $\qquad x \in (0,\pi)+k\pi,\ k\in\mathbb{Z}$



- $\cos x \neq 1$ $\qquad x \neq 0+k\pi$

  $\cos x \neq -1$ :)

- (1) $\frac{\sin x}{1-(-\cos x)^2} = \frac{\sin x}{1-\cos^2 x} \overset{?}{\cancel{=}} -\frac{\sin x}{1-\cos^2 x}$ :(

  (2) $\frac{-\sin x}{1-\cos^2 x} \overset{?}{=} -\frac{\sin x}{1-\cos^2 x}$ ✓ :)

  $t=\cos x$

  (3) $\frac{-\sin x}{1-(-\cos x)^2} \overset{?}{=} \frac{\sin x}{1-\cos^2 x}$ ✗ :(

- $t=\cos x \qquad dt = -\sin x\,dx$

  $f(t) \quad t \neq \pm 1$

$$\int \frac{-\sin x}{1-\cos^2 x}\,dx = \int \overbrace{\frac{-1}{1-t^2}}^{f(t)}\,dt =$$

$$\overset{c}{=} -\frac{1}{2}\ln\left|\frac{1+t}{1-t}\right| = -\frac{1}{2}\ln\left|\frac{1+\cos x}{1-\cos x}\right|$$

- 1. VoS $\qquad \varphi(0,\pi)=(-1,1)$

  $\varphi=\cos x \qquad (\alpha,\beta)=(0,\pi)+k\pi$

  $f=\frac{-1}{1-t^2}$ $\qquad \cancel{(-\infty,-1)}$

  $(-1,1)=(a,b)$

  $\cancel{(1,\infty)}$

  $\varphi(\alpha,\beta)\subseteq(a,b)$

  Záver: $x\in(\alpha,\beta)=(0,\pi)+k\pi$

$$\int \frac{1}{\sin^2 x + 2\cos^2 x}\, dx$$

- cíl $x \in \mathbb{R}$

- Test

(3) $\displaystyle \frac{1}{(-\sin x)^2 + 2(-\cos x)^2} \overset{\checkmark}{=} \frac{1}{\sin^2 x + 2\cos^2 x}$

$t = \operatorname{tg} x$ ✓ $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi$

- $\displaystyle dt = \frac{1}{\cos^2 x}\, dx$ $\quad \left( \operatorname{arctan} t = x \quad \frac{1}{1+t^2}\, dt = dx \right)$

$$\int \frac{1}{\sin^2 x + 2\cos^2 x} \cdot \cos^2 x \cdot \underbrace{\frac{1}{\cos^2 x}\, dx}_{} =$$

$$= \int \frac{1}{\frac{t^2}{1+t^2} + 2 \cdot \frac{1}{1+t^2}} \cdot \frac{1}{1+t^2}\, dt$$

$$= \int \underbrace{\frac{1}{t^2+2}}_{f}\, dt = \int \frac{1}{2\left(1+\left(\frac{t}{\sqrt{2}}\right)^2\right)}\, dt$$

$$\overset{c}{=} \frac{1}{2}\sqrt{2}\, \operatorname{arctan} \frac{t}{\sqrt{2}} = \frac{\sqrt{2}}{2} \operatorname{arctan}\left(\frac{\tan x}{\sqrt{2}}\right)$$

1.VoS.

$\varphi = \tan x \qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi$ $\quad (\alpha, \beta)$

$\displaystyle f = \frac{1}{2+t^2} \qquad t \in \mathbb{R} = (a, b)$

$\varphi(\alpha, \beta) = \mathbb{R} \subseteq (a, b)$ ✓ Lepení (na delší)

$\int$ našli na $\left(-\pi/2, \pi/2\right) + k\pi$