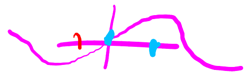
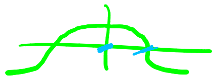
$$\int \frac{\overbrace{\sqrt{9-x^2}}^{f}}{x^2}\,dx \qquad 1=\cos^2 t+\sin^2 t \qquad f:\ \begin{matrix}(0,3)\\(a,b)\end{matrix} \qquad \begin{matrix}(-3,0)\\(a_1,b_1)\end{matrix} \to (\alpha_1,\beta_1)=(-\tfrac{\pi}{2},0)$$

$\arcsin\frac{x}{3} = t$ $\qquad \varphi^{-1}(x) = \arcsin\frac{x}{3} \qquad (\alpha,\beta)=(0,\frac{\pi}{2})$

$x = 3\sin t \qquad \varphi(t) = 3\sin t \qquad \varphi(0,\pi/2) = (0,3)$

$dx = 3\cos t\,dt \qquad \varphi'(t) = 3\cos t \qquad 3\cos t \neq 0$ na $(0,\pi/2)$

$$\int \overbrace{\frac{\sqrt{9-9\sin^2 t}}{9\sin^2 t}\cdot 3\cos t\,dt}^{f(\varphi)\cdot\varphi'} = \int \frac{\sqrt{9}\sqrt{1-\sin^2 t}}{3\sin^2 t}\cos t\,dt =$$

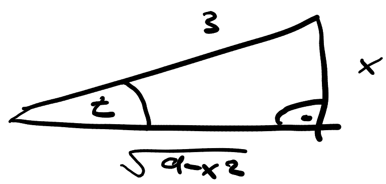
$$= \int \frac{\sqrt{\cos^2 t}}{\sin^2 t}\cos t\,dt \overset{!}{=} \int \frac{\cos t}{\sin^2 t}\cos t\,dt = \int \frac{1-\sin^2 t}{\sin^2 t}\,dt$$

$$= \int \frac{1}{\sin^2 t} - 1\,dt \overset{c}{=} \underbrace{-\cot g\, t - t}_{G(t)} = \underbrace{-\cot\left(\arcsin\tfrac{x}{3}\right) - \arcsin\tfrac{x}{3}}_{G(\varphi^{-1}(x))}$$

$x\in(a,b) \to x\in(0,3)$

$x\in(-3,0)$

$$= -\frac{\sqrt{9-x^2}}{x} - \arcsin\frac{x}{3}$$



$x = 3\sin t$

$\cot t = \frac{\sqrt{9-x^2}}{x}$

Vsuvka

$$1 = \cos^2 x + \sin^2 x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\int \cos^2 x\,dx \qquad \frac{1+\cos 2x}{2} = \cos^2 x$$

$$\overset{c}{=} \frac{1}{2}x + \frac{1}{2}\cdot\frac{\sin 2x}{2}$$

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f $\qquad$ $f: x \in \mathbb{R} = (a,b)$

$$\int \sqrt{4+x^2}\, dx$$

$\operatorname{argsinh} \frac{x}{2} = t$ $\qquad$ $\varphi(t)$

$x = 2 \sinh t$ $\qquad$ $(\alpha,\beta) = \mathbb{R}$ $\qquad$ $\varphi(\mathbb{R}) = \mathbb{R}$

$dx = 2\cosh t\, dt$ $\qquad$ $2\cosh t \neq 0$ $\qquad$ $t \in \mathbb{R}$

$$\int \sqrt{4+4\sinh^2 t} \cdot 2\cosh t\, dt = 2\int \sqrt{4}\sqrt{1+\sinh^2 t}\cosh t\, dt$$

$$= \int 4 \sqrt{\cosh^2 t} \cdot \cosh t\, dt = 4\int \cosh^2 t\, dt$$

$|\cosh t|$

$$\overset{c}{=} 2\left(t + \frac{1}{2}\sinh(2t)\right)$$

$$= 2\operatorname{argsinh}\frac{x}{2} + \underline{\sinh}\left(2\operatorname{argsinh}\frac{x}{2}\right)$$

$x \in \mathbb{R}$

$$\int \frac{1}{1+e^x} \cdot \frac{e^x}{e^x}\, dx \qquad \int \frac{2\sqrt{y}}{1+\sqrt{y}} \cdot \frac{1}{2\sqrt{y}}\, dy$$