Lepení

$$f = \begin{cases} 3x^2 & x > 0 \\ -3x^2 & x \le 0 \end{cases}$$

- f spoj. na $\mathbb{R}$ $\longrightarrow$ PF bude na $\mathbb{R}$

- $$F = \begin{cases} x^3 + c & x \in (0, \infty) \\ -x^3 + d & x \in (-\infty, 0) \end{cases}$$

- PF je spoj.,

$x = 0$:

$$\lim_{x \to 0-} F(x) = \lim_{x \to 0+} F(x)$$

$$\lim_{x \to 0-} -x^3 + d = \lim_{x \to 0+} x^3 + c$$

$$\boxed{d = c}$$

Závěr:

- $$F(x) = \begin{cases} x^3 + c & x > 0 \\ c & x = 0 \\ -x^3 + c & x < 0 \end{cases}$$

$$f = |\cos x|$$

- f spoj. na $\mathbb{R}$ $\longrightarrow$ F bude na $\mathbb{R}$

- $$f = \begin{cases} \cos x & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + 2k\pi \\ -\cos x & x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) + 2k\pi \end{cases}$$

$$F = \begin{cases} \sin x + c_k & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + 2k\pi \\ -\sin x + d_k & x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) + 2k\pi \end{cases}$$

$e^{-|x|}$

$x = 0$

$x < 0$ | $x > 0$

$\int e^x = e^x + c$ | $\int e^{-x} = -e^{-x} + d$

$\lim_{x \to 0-} e^x + c$ | $\lim_{x \to 0+} -e^{-x} + d$

$1 + c = -1 + d$

$c = -2 + d$

$c + 2 = d$

$e^x + -2 + d$ $\quad -1+d \quad$ $-e^{-x} + d$

$e^x + c$ $\quad 1+c \quad$ $-e^{-x} + c + 2$

$\sin x + c_{-1}$ $\qquad$ $\sin x + c_0$ $\qquad$ $\sin x + c_1$

$-\frac{3\pi}{2}$ $\quad$ $-\frac{\pi}{2}$ $\quad$ 0 $\quad$ $\frac{\pi}{2}$ $\quad$ $\frac{3\pi}{2}$ $\quad$ $\frac{5\pi}{2}$

$-\sin x + d_{-1}$ $\qquad$ $-\sin x + d_0$

$-\frac{\pi}{2}$:

$$\lim_{x \to -\frac{\pi}{2}-} F(x) = \lim_{x \to -\frac{\pi}{2}+} F(x)$$

c_0 :)

$$\lim_{x \to -\frac{\pi}{2}-} -\sin x + d_{-1} = \lim_{x \to -\frac{\pi}{2}+} \sin x + c_0$$

$$1 + d_{-1} = -1 + c_0$$

$$d_{-1} = c_0 - 2$$

$\frac{\pi}{2}$:

$$\lim_{x \to \frac{\pi}{2}-} \sin x + c_0 = \lim_{x \to \frac{\pi}{2}+} -\sin x + d_0$$

$$1 + c_0 = -1 + d_0$$

$$2 + c_0 = d_0$$

$\frac{3\pi}{2}$

$$\lim_{x \to \frac{3\pi}{2}-} -\sin x + d_0 = \lim_{x \to \frac{3\pi}{2}+} \sin x + c_1$$

$$1 + d_0 = -1 + c_1$$

$$2 + d_0 = c_1$$

$$2 + 2 + c_0 = c_1$$

$$4 + c_0 = c_1$$

$$c_k = c_0 + 2k$$

$$d_k = c_0 + 4k + 2$$

$$F = \begin{cases} \sin x + c_0 + 4k & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + 2k\pi \\ c_0 + 2k + 1 & x = \frac{\pi}{2} + 2k\pi \\ -\sin x + c_0 + 4k + 2 & x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) + 2k\pi \end{cases}$$