

Lebesgue

$$f = \begin{cases} 3x^2 & x > 0 \\ -3x^2 & x \leq 0 \end{cases}$$

• f spoj. na \mathbb{R} \rightarrow PF bucle na \mathbb{R}

$$F = \begin{cases} x^3 + c & x \in (0, \infty) \\ -x^3 + d & x \in (-\infty, 0) \end{cases}$$

• PF je spoj. ,

$x=0$:

$$\lim_{x \rightarrow 0^-} F(x) = \lim_{x \rightarrow 0^+} F(x)$$

$$\lim_{x \rightarrow 0^-} -x^3 + d = \lim_{x \rightarrow 0^+} x^3 + c$$

$$\boxed{d = c}$$

Záver:

$$F(x) = \begin{cases} x^3 + c & x > 0 \\ c & x = 0 \\ -x^3 + c & x < 0 \end{cases}$$

$$f = |\cos x|$$

• f spoj. na \mathbb{R} \rightarrow F bucle na \mathbb{R}

$$f = \begin{cases} \cos x & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + 2k\pi \\ -\cos x & x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) + 2k\pi \end{cases}$$

$$F = \begin{cases} \sin x + c_1 & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + 2k\pi \\ -\cos x + c_2 & x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) + 2k\pi \end{cases}$$

$$e^{-|x|}$$

$$x < 0$$

$$\int e^x = e^x + c$$

$$\lim_{x \rightarrow 0^-} e^x + c$$

$$1 + c$$

$$c$$

$$e + 2$$

$$e^x + -2 + d$$

$$e^x + c$$

$$x = 0$$

$$x > 0$$

$$\int e^{-x} = -e^{-x} + d$$

$$\lim_{x \rightarrow 0^+} -e^{-x} + d$$

$$-1 + d$$

$$= -2 + d$$

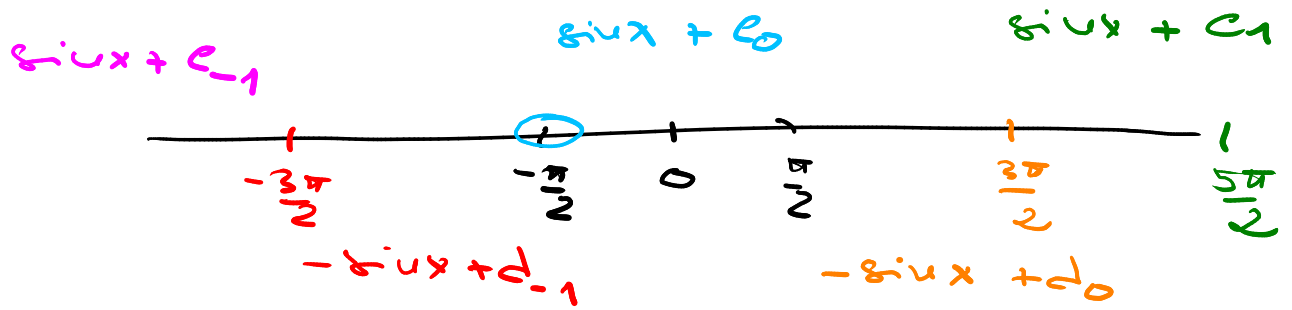
$$= d$$

$$-1 + d$$

$$1 + c$$

$$-e^{-x} + d$$

$$-e^{-x} + c + 2$$



$-\frac{\pi}{2}$:

$$\lim_{x \rightarrow -\frac{\pi}{2}^-} F(x) = \lim_{x \rightarrow -\frac{\pi}{2}^+} F(x) \quad c_0 \ddot{}$$

$$\lim_{x \rightarrow -\frac{\pi}{2}^-} -\sin(x) + d_{-1} = \lim_{x \rightarrow -\frac{\pi}{2}^+} \sin(x) + c_0$$

$$1 + d_{-1} = \underbrace{-1 + c_0}$$

$$d_{-1} = c_0 - 2$$

$\frac{\pi}{2}$:

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \sin(x) + c_0 = \lim_{x \rightarrow \frac{\pi}{2}^+} -\sin(x) + d_0$$

$$1 + c_0 = \underbrace{-1 + d_0}$$

$$\underbrace{2 + c_0} = \underbrace{d_0}$$

$\frac{3\pi}{2}$:

$$\lim_{x \rightarrow \frac{3\pi}{2}^-} -\sin(x) + d_0 = \lim_{x \rightarrow \frac{3\pi}{2}^+} \sin(x) + c_1$$

$$\underbrace{1 + d_0 = -1 + c_1}$$

$$2 + d_0 = c_1$$

$$2 + 2 + c_0 = c_1$$

$$4 + c_0 = c_1$$

$$c_2 = c_0 + 2k$$

$$d_2 = c_0 + 2k + 2$$

$$f = \begin{cases} \sin x + c_0 + 2k & x \in (-\frac{\pi}{2}, \frac{\pi}{2}) + 2\pi \\ c_0 + 2k + 1 & x = \frac{\pi}{2} + 2\pi \\ -\sin x + c_0 + 2k + 2 & x \in (\frac{\pi}{2}, \frac{3\pi}{2}) + 2\pi \end{cases}$$