

Limity funkcí I

1. Dokažte z definice, že

$$\text{a) } \lim_{x \rightarrow 1} \left(\frac{x}{2}\right)^3 = \frac{1}{8}$$

$$\text{b) } \lim_{x \rightarrow 1^+} [x] = 1$$

$$\text{c) } \lim_{x \rightarrow 1^-} [x] = 0$$

Spočtěte

$$2. \text{ (a) } \lim_{x \rightarrow 0} \frac{x^2 - 1}{2x^2 - x - 1} \quad \text{(b) } \lim_{x \rightarrow 1} \frac{x^2 - 1}{2x^2 - x - 1}$$

$$3. \lim_{x \rightarrow 2} \left(\frac{1}{x^2 - 2x} - \frac{x}{x^2 - 4} \right)$$

$$4. \lim_{x \rightarrow 0} \frac{(1+x)(1+2x)\dots(1+nx) - 1}{x}, n \in \mathbb{N}$$

$$5. \lim_{x \rightarrow 1} \frac{x^{100} - 2x + 1}{x^{50} - 2x + 1}$$

$$6. \lim_{x \rightarrow 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}, m, n \in \mathbb{N}$$

$$7. \lim_{x \rightarrow 1} \frac{x^{n+1} - (n+1)x + n}{(x-1)^2}, n \in \mathbb{N}$$

$$8. \lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x - 1}, n \in \mathbb{N}$$

$$9. \lim_{x \rightarrow 1} \left(\frac{m}{1-x^m} - \frac{n}{1-x^n} \right), m, n \in \mathbb{N}$$

$$10. \lim_{x \rightarrow 0} \frac{\frac{2}{x^2} + 1}{\sqrt{\frac{3}{x^4} - \frac{6}{x^2} + 5}}$$

$$11. \lim_{x \rightarrow 0^+} \frac{(\sqrt{\frac{1}{x^2} + 1} - \sqrt{\frac{1}{x^2} - 1})}{x}$$

$$12. \lim_{x \rightarrow 0^+} \left(\sqrt{\frac{1}{x} + \sqrt{\frac{1}{x} + \sqrt{\frac{1}{x}}}} - \sqrt{\frac{1}{x} - \sqrt{\frac{1}{x} + \sqrt{\frac{1}{x}}}} \right)$$

13. (a) $\lim_{x \rightarrow 16} \frac{\sqrt[4]{x} - 2}{\sqrt{x} - 4}$ (b) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$
14. $\lim_{x \rightarrow 0} \frac{\sqrt{1 - 2x - x^2} - (1 - x)}{x}$
15. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{27+x} - \sqrt[3]{27-x}}{x + 2\sqrt[3]{x^4}}$
16. $\lim_{x \rightarrow 0} \frac{\sqrt[m]{1+x} - \sqrt[n]{1+x}}{x}, m, n \in \mathbb{N}$
17. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt[3]{1-x}}{\sqrt[3]{1+x} - \sqrt{1-x}}$
18. $\lim_{x \rightarrow a^+} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2 - a^2}}, a \in \mathbb{R}_0^+$
19. $\lim_{x \rightarrow 0} \frac{\sqrt[m]{1+ax} \sqrt[n]{1+bx} - 1}{x}, m, n \in \mathbb{N}, a, b \in \mathbb{R}$