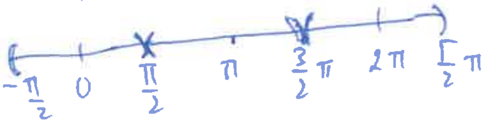
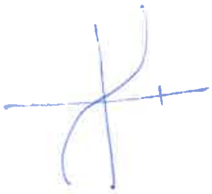
$$\int \frac{\sin^2 x}{1+\sin^2 x}\,dx = \dots = \left| \begin{matrix} y=\tan x \\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right) + k\pi \end{matrix}\right| = \dots \int \frac{dy}{1+y^2} + \frac{-1}{1+2y^2}\,dy$$

$$= \operatorname{arctg} y - \frac{1}{\sqrt{2}}\operatorname{arctg} \sqrt{2}\, y + \tilde{C}, \quad \tilde{C} = C_k$$

$$= x - \frac{1}{\sqrt{2}} \arctan\left(\sqrt{2}\tan x\right) + C_k$$





$$\lim_{x\to\left(\frac{\pi}{2}+k\pi\right)^-} \left(x - \frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}\tan x\right) + C_k\right) = \lim_{x\to\left(\frac{\pi}{2}+k\pi\right)^+} \left(x - \frac{1}{\sqrt{2}}\operatorname{at}\left(\sqrt{2}\tan x\right) + C_{k+1}\right)$$

$$\cancel{\frac{\pi}{2}+k\pi} - \frac{1}{\sqrt{2}}\frac{\pi}{2} + C_k = \cancel{\frac{\pi}{2}+k\pi} + \frac{1}{\sqrt{2}}\frac{\pi}{2} + C_{k+1}$$

$$C_{k+1} - C_k = \frac{-1}{\sqrt{2}}\pi$$

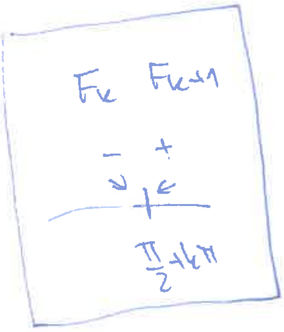
$C_0 := C \in \mathbb{R}$ libovolné

$$C_1 = C - \frac{\pi}{\sqrt{2}}$$

$$C_2 = \left(C - \frac{\pi}{\sqrt{2}}\right) - \frac{\pi}{\sqrt{2}} = C - 2\frac{\pi}{\sqrt{2}}$$

$$\vdots$$

$$C_k = C - \frac{\pi k}{\sqrt{2}}, \quad k \in \mathbb{Z}.$$



$$F(x) = \begin{cases} F_k(x) = x - \frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}\tan x\right) + C - \frac{\pi}{\sqrt{2}}k & x \in \left(-\frac{\pi}{2}+k\pi, \frac{\pi}{2}+k\pi\right) \\ x = \frac{\pi}{2}+k\pi : \lim_{x\to\left(\frac{\pi}{2}+k\pi\right)^-} F_k = \frac{\pi}{2}+k\pi - \frac{\pi}{2\sqrt{2}} + C - \frac{\pi}{\sqrt{2}}k \end{cases}$$

Zk.:

$$\lim_{x\to\left(\frac{\pi}{2}+k\pi\right)^-} F_k = \boxed{\frac{\pi}{2}+k\pi - \frac{1}{\sqrt{2}}\frac{\pi}{2} + C - \frac{\pi}{\sqrt{2}}k}$$

$$\|$$

$$\lim_{x\to\left(\frac{\pi}{2}+k\pi\right)^+} F_{k+1} = \frac{\pi}{2}+k\pi + \frac{1}{\sqrt{2}}\frac{\pi}{2} + C - \frac{\pi}{\sqrt{2}}(k+1)$$

1. Nechť $f(x) = \sqrt{1-e^{-x^2}} \arctan\left(\frac{x}{x+1}\right)$.

Určete definiční obor D_f.

Spočtěte derivaci.

Napište obor, kde Váš výpočet platí.

BONUS: Spočtěte $f'(0)$, pokud existuje.

2. Určete $\int \frac{1}{\sin^2 x + 2\sin x \cos x + 2\cos^2 x} dx$ (Použijte $y = \operatorname{tg} x$)

Nevl. limity a limity v nevl. bodech

(A)

① $c \pm \infty = \pm\infty$, $c > -\infty$
$c - \infty = -\infty$, $c < \infty$, $c \in \mathbb{R}^* = \mathbb{R}\cup\{\pm\infty\}$! $+\infty - \infty$ NDF

② $c\cdot \pm\infty = \pm\infty$, $c>0$
$c\cdot\pm\infty = \mp\infty$, $c<0$, $c\in\mathbb{R}^* := \mathbb{R}\cup\{\pm\infty\}$! $0\cdot\infty$ NDF
Spc $c \neq 0$

③ $\frac{1}{\pm\infty} = 0^\pm$ $\frac{1}{0^\pm} = \pm\infty$

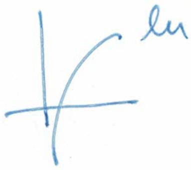
(odtud třeba $\frac{c}{+\infty}$, $c>0$, $c\in\mathbb{R}$; $\frac{c}{\frac{1}{0^+}} = c\cdot 0^+ = 0$.)

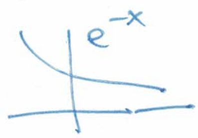
NDF: $\frac{1}{0}$!  $\lim \frac{1}{x} \nexists$ (Ani ndf $\frac{\pm\infty}{0}$!) Ale $\frac{+\infty}{+\infty} = +\infty\cdot\frac{1}{0^+} = +\infty$

Viz před. Dál třeba $\frac{c>0}{0^\pm} = c\cdot\frac{1}{0^\pm} = c\cdot\pm\infty = \pm\infty$ (vč. $c=+\infty$)

④ Moc.: NDF 1^∞ převádím na exp. → V limitách občasná (ale typická!) chyba je: $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty} 1^x = 1$! (má být e) TZV. ČÁST. DOSAZENÍ.

Pozn.: $(0^+)^{+\infty} = e^{(\ln 0^+)(+\infty)} = e^{-\infty\cdot+\infty} = e^{-\infty} = 0$. Velmi symbolický výpočet. Nemáme definováno.





(C) Růstová řada

$a^x \gg x^\alpha \gg \log x$ u $+\infty$. Tím myslíme
$a>1$ $\alpha>0$

$\lim_{x\to+\infty}\frac{a^x}{x^\alpha} = +\infty \wedge \lim_{x\to+\infty}\frac{x^\alpha}{\log x} = +\infty$. Co to je vlastně $\lim_{x\to\pm\infty}$?

(B) Limity v nevlastních bodech

$$\lim_{x\to\pm\infty} f(x) := \lim_{x\to 0^\pm} f\left(\frac{1}{x}\right) \left(= \lim_{y\to 0^\pm} f\left(\frac{1}{y}\right)\right)$$

Nutné $f\circ\frac{1}{x}$ def. na $P^+_\delta(0)$ nebo na $P^-_\delta(0)$

$P^\pm_\delta(a) := \{x\in\mathbb{R} \mid x \gtrless a\}\cap P_\delta(a)$

levé a pravé prstencové okolí

Např. $P^-_\delta(a) = \{x\in\mathbb{R}\mid a-\delta<x<a\}$

(D) Podrobnější růstové řady:

$$e^{-x} \ll x^{-\beta} \ll x^{-\alpha} \ll \frac{1}{\log x} \ll 1 \ll \log x \ll x^{\alpha} \ll x^{\beta} \ll e^{x}$$

$0 < \alpha < \beta \qquad x \to +\infty$

$$x \to 0^+: \quad x^{\beta} \ll x^{\alpha} \ll \frac{1}{\log \frac{1}{x}} \ll 1 \ll \log \frac{1}{x} \ll x^{-\alpha} \ll x^{-\beta}$$

$$\left(\frac{1}{-\log x}\right) \qquad (-\log x)$$

Mocniny na exp: princip $0.1^x \to 0$, ale $10^x \to \infty$.

(E) Limity posloupnosti $\lim_{n \to +\infty} a_n = A \Leftrightarrow \forall \varepsilon > 0\ \exists n_0\ \forall n \geq n_0$ $|a_n - A| < \varepsilon$. Analogicky jako pro fce $\lim_{n \to \infty} a_n = \pm\infty$.

Heine: $f: \mathbb{R} \to \mathbb{R}^*$, $a \in \mathbb{R}^*$, $A \in \mathbb{R}^*$, $\forall \delta: D_f \cap P_\delta(a) \neq \emptyset$. $\lim_{x \to a} f(x) = A \Leftrightarrow \forall (x_n) \subseteq D_f \setminus \{a\}\ x_n \to a$ je $\lim_{n \to \infty} f(x_n) = A$

Př.: Limity v nevlast. bodech

1. Základní $\displaystyle \lim_{x \to +\infty} \frac{x^3 + 2x + 1}{x^5 + 3x + 7} = \lim_{x \to 0^+} \frac{\frac{1}{x^3} + \frac{2}{x} + 1}{\frac{1}{x^5} + \frac{3}{x} + 7} =$ ODSTR. ZLOMKŮ

$$= \lim_{x \to 0^+} \frac{x^2 + 2x^4 + x^5}{1 + 3x^4 + 7x^5} = \frac{0}{1} = 0$$

Základní $\displaystyle \lim_{x \to +\infty} \frac{x^4 + 2x + 2}{13x^4 + x + 7} = \lim_{x \to 0^+} \frac{\frac{1}{x^4} + \frac{2}{x} + 2}{\frac{13}{x^4} + \frac{1}{x} + 7} = \lim_{x \to 0^+}$ $\neq \frac{2}{7}$! nýbrž

$$\frac{1 + 2x^3 + 2x^4}{13 + x^3 + 7x^4} = \frac{1 + 0 + 0}{13 + 0 + 0} = \frac{1}{13}$$

Zákl. $\displaystyle \lim_{x \to +\infty} \frac{x^4 + x^2}{x^2 + x^3}$ analogicky: $\displaystyle \lim_{x \to 0^+} \frac{\frac{1}{x^4} + \frac{1}{x^2}}{\frac{1}{x^2} + \frac{1}{x^3}} = \lim_{x \to 0^+} \frac{1 + x^2}{x^2 + x} =$

$$= \text{"}\frac{1}{0^+}\text{"} = +\infty$$

"Aritmetika limit"

Př.: Obdobně spočteme:

$$\lim_{x\to+\infty} \frac{2x^2+1}{\sqrt{3x^4-6x^2+5}} = \lim_{x\to 0^+} \frac{\frac{2}{x^2}+1}{\sqrt{\frac{3}{x^4}-\frac{6}{x^2}+5}} =$$

$$= \lim_{x\to 0^+} \frac{2+x^2}{\sqrt{3-6x^2+5x^4}} = \frac{2}{\sqrt{3}}.$$

Př.: $$\lim_{x\to\infty} x\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right) = \lim_{x\to 0^+} \frac{\sqrt{\frac{1}{x^2}+1}-\sqrt{\frac{1}{x^2}-1}}{x} =$$

$$= \lim_{x\to 0^+} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x^2} \overset{\frac{0(+)}{0(+)}\text{ NDF}}{=} \lim_{x\to 0^+} \frac{\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}$$

$$= \lim_{x\to 0^+} \frac{2x^2}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)} = \lim_{x\to 0^+} \frac{2}{\sqrt{1+x^2}+\sqrt{1-x^2}} = \frac{2}{2} = 1$$

L'Hospitalovo pravidlo

Předpoklad

1. f', g' vlastní, $g' \neq 0$ na $P_\delta(a)$ $(\exists \delta)$ — $(a-\delta, a+\delta)\setminus\{a\}$
2. $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$ nebo

   $\lim\limits_{x\to a} f(x) = \pm\infty$, $\lim\limits_{x\to a} g(x) = \pm\infty$.
3. Existuje $\lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$.

Pak $\exists \lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$

Pozn.: $\frac{0}{\infty}$ lze také spočítat l'Hosp.

Př.: $\lim\limits_{x\to+\infty} \sqrt[x]{x} = \lim\limits_{x\to+\infty} x^{\frac{1}{x}} = \lim\limits_{x\to+\infty} \underline{e^{\frac{1}{x}\ln x}} = \lim\limits_{x\to 0^+} e^{x\ln\frac{1}{x}} = \lim\limits_{x\to 0^+} e^{\overline{-x\ln x}}$

$\left[\lim\limits_{x\to 0^+}(-\underline{x\ln x}) = -0 \right.$ $\left| = \lim\limits_{x\to 0^+} e^{-x\ln x} = \underline{e^{-0^+} = 1.}\right]$

↑ růstová řada

Podrobněji: $0^+ \leftarrow \dfrac{+x}{\frac{1}{-\ln x}} = -x\ln x$ $\left[x \ll \dfrac{1}{-\ln x} \Rightarrow \dfrac{\frac{1}{-\ln x}}{x} \to +\infty \quad \dfrac{x}{\frac{1}{-\ln x}} \to 0^+ \right]$

Př.: Růstové řady

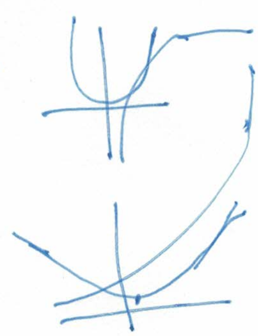
$n \geq 0$ $\lim\limits_{x\to\infty} \dfrac{x^n}{e^x} = \left|\dfrac{\infty}{\infty}\right| \overset{\text{l'Hosp.}}{=} \lim\limits_{x\to\infty} \dfrac{n x^{n-1}}{e^x} = \dots = \lim\limits_{x\to\infty} \dfrac{n!}{e^x} = 0$

$\left(= n! \lim\limits_{x\to\infty} e^{-x} = n!\ 0 = 0 \right)$

Pro $n < 0$ Varit m. limit

Př.: $\lim\limits_{x\to\infty} \dfrac{x^n}{\ln x} = \lim \dfrac{n x^{n-1}}{\frac{1}{x}} = \lim\limits_{x\to\infty} n x^n = +\infty$, $n > 0$

Zde je dobré: do paměti „obrázky“:



(l'Hosp. je odůvodnění rychlost růstu)

L'Hospital (h, s nečte l-hvaže!)

Př.: $\lim\limits_{x\to 0} \frac{\operatorname{tg} x - x}{x - \sin x} \overset{l'H}{\underset{\frac{0}{0}\ \text{Lze}}{=}} \lim\limits_{x\to 0} \frac{\frac{1}{\cos^2 x} - 1}{1 - \cos x} = \lim\limits_{x\to 0} \frac{1 - \cos^2 x}{(1-\cos x)\cos^2 x} =$

$= \lim\limits_{x\to 0} \frac{1+\cos x}{\cos^2 x} = \frac{2}{1} = 2$

Př.: $\lim\limits_{x\to 0} \frac{x(e^x+1) - 2(e^x-1)}{x^3} \overset{l'H}{=} \lim\limits_{x\to 0} \frac{e^x + 1 + xe^x - 2e^x}{3x^2} =$

$= \lim\limits_{x\to 0} \frac{xe^x - e^x + 1}{3x^2} \overset{l'H}{\underset{\frac{0}{0}}{=}} \lim\limits_{x\to 0} \frac{e^x + xe^x - e^x}{6x} = \lim\limits_{x\to 0} \frac{e^x + xe^x}{6} = \frac{1}{6}.$

Př.: $\lim\limits_{x\to 0} \dfrac{1-\cos^2 x}{x^2 \sin x^2} \overset{lH}{=} \lim\limits_{x\to 0} \dfrac{2x \sin x^2}{2x\sin x^2 + x^2 2x \cos x^2} = \lim\limits_{x\to 0} \dfrac{\sin x^2}{\sin x^2 + x^2 \cos x^2} =$

$= \lim\limits_{x\to 0} \dfrac{\frac{\sin x^2}{x^2}}{\frac{\sin x^2}{x^2} + \cos x^2} \overset{\text{vos}}{=} \dfrac{1}{1+1} = \dfrac{1}{2}.$

(vosem na $\dfrac{\sin y}{y}$)

Př.: $\lim\limits_{x\to 0^+} x^x = \lim\limits_{x\to 0^+} e^{x\ln x} = e^{\lim\limits_{x\to 0} x \ln x} \underset{\text{Rušt.}}{=} e^0 = 1$

Mistorušku lze i $\lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}} = \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0^+} x = 0.$

Př.: $\lim\limits_{x\to \frac{\pi}{4}} (\operatorname{tg} x)^{\operatorname{tg} 2x} = e^{\lim\limits_{x\to\frac{\pi}{4}} \operatorname{tg} 2x \ln(\operatorname{tg} x)}$

$\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\ln(\operatorname{tg} x)}{\frac{1}{\operatorname{tg} 2x}} = \lim\limits_{x\to\frac{\pi}{4}} \dfrac{\frac{1}{\operatorname{tg} x}\,\frac{1}{\cos^2 x}}{-\frac{1}{\operatorname{tg}^2 2x}\,\frac{1}{\cos^2 2x}\cdot 2} =$

$= \lim\limits_{x\to\frac{\pi}{4}} \dfrac{-\sin^2 2x}{2\operatorname{tg} x \cos^2 x} = -\dfrac{1}{2\cdot 1 \left(\frac{\sqrt{2}}{2}\right)^2} = -1 \Rightarrow \lim\limits_{x\to\frac{\pi}{4}} (\operatorname{tg} x)^{\operatorname{tg} 2x} = \dfrac{1}{e}$

Symboly o, O, $\sim$

1. „$f(x) = o(g(x)),\ x \to a$" $\equiv \lim_{x\to a} \frac{f(x)}{g(x)} = 0$

$x^n = o(x^{n-1}),\ x \to 0$? $\quad \lim_{x\to 0} \frac{x^n}{x^{n-1}} = \lim_{x\to 0} x = 0$

$x^n = o(x^{n+1}),\ x \to \infty$? $\quad \lim_{x\to\infty} \frac{x^n}{x^{n+1}} = \lim_{x\to\infty} \frac{1}{x} = 0 \left(= \lim_{y\to 0} \frac{1}{\frac{1}{y}} = \lim_{y\to 0} y = 0\right)$

2. „$f(x) = O(g(x)),\ x \to a$" $\equiv \exists C > 0\ \exists \delta > 0\ \forall x \in P_\delta(a)\ |f(x)| \leq C|g(x)|$

(stačí $\lim_{x\to a} \left|\frac{f(x)}{g(x)}\right| \in \mathbb{R} \setminus \{0\}$, není ale nutné.)

3. „$f(x) \sim g(x),\ x \to a$" $\equiv \lim_{x\to a} \frac{f(x)}{g(x)} = 1$ asympt. ekv. &

Někdy: „$f(x) \sim g(x),\ x \to a$" $\equiv \lim_{x\to a} \frac{f(x)}{g(x)} \in \mathbb{R} \setminus \{0\}$ & slabě asym. ekv.

„$f(x) \simeq g(x),\ x \to a$" $\equiv \lim_{x\to a} \frac{f(x)}{g(x)} = 1$" &

Př.: $e^x - \cos x \sim x$, $x \to 0$ Dk.!

$$\lim_{x\to 0} \frac{e^x - \cos x}{x} = \lim_{x\to 0} \frac{e^x + \sin x}{1} = 1$$

Př.: $e^x - \cos x = o(x^a)$, $x \to 0$ $\forall a < 1$

$$\lim_{x\to 0} \frac{e^x - \cos x}{x^a} = \lim_{x\to 0} \frac{e^x - \cos x}{x} x^{1-a} = 1 \lim_{x\to 0} x^{1-a} = 1 \cdot 0 = 0.$$

Př. Limita posloupnosti

1.
$$\lim_{n\to\infty} \frac{\sqrt{n^3 - 2n^2 + 1} + \sqrt[3]{n^4+1}}{\sqrt{n^6 - 6n^5 + 2} + \sqrt[5]{n^7 + n^3 + 1}} \overset{\text{triv. Heine, "neodef"}}{=} \lim_{x\to\infty} \frac{\sqrt{x^3 - 2x^2 + 1} + \sqrt[3]{x^4+1}}{\sqrt{x^6 - 6x^5 + 1} + \sqrt[5]{x^7 + x^3 + 1}}$$

$\boxed{\tfrac{3}{2}}$ $\boxed{\tfrac{4}{3}}$ $\boxed{3}$ $\boxed{7/5}$ míňus 0?

$$= \lim_{x\to 0^+} \frac{\sqrt{\frac{1}{x^3} - \frac{2}{x^2} + 1} + \sqrt[3]{\frac{1}{x^4} + 1}}{\sqrt{\frac{1}{x^6} - \frac{6}{x^5} + 1} + \sqrt[5]{\frac{1}{x^7} + \frac{1}{x^3} + 1}} \cdot \frac{x^3}{x^3} =$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x^3 - 2x^4 + x^6} + \sqrt{x^5 + x^9}}{\sqrt{1 - 6x + x^6} + \sqrt{x^8 + x^{12} + x^{15}}} = \frac{0+0}{1+0} = 0$$

2. $\lim_{n\to\infty} \frac{a^n}{n!}$, $a \in \mathbb{R}$ $\quad \lim_{n\to\infty} \frac{a^n}{n!} = \lim_{n\to\infty} \frac{a}{n} \frac{a}{n-1} \cdots \frac{a}{1}$, Zvol ε.

Najdi $n_0 \in \mathbb{N}$, že $\frac{a}{n_0} < \varepsilon$ $\quad \forall n \geq n_0$ $\frac{a}{n} < \varepsilon$

Prehlednejsi obecnejsi postup na tabuli. Zde totiz potrebujeme n sude, a>0 a eps.< 1.

$$= \lim_{n\to\infty} \left(\underbrace{\frac{a}{1} \cdots \frac{a}{n_0 - 1}} \cdot \underbrace{\frac{a}{n_0+1} \cdots \frac{a}{n}}_{\text{triv. Heine}} \right) \leq \lim_{n\to\infty} \left(\frac{a}{1} \cdots \frac{a}{n_0 - 1} \right) \cdot \varepsilon^{n - n_0} = 0$$

3. $$\lim_{n\to\infty} \sqrt[n]{n} = \lim_{n\to\infty} n^{\frac{1}{n}} \overset{\checkmark}{=} \lim_{x\to\infty} x^{\frac{1}{x}} = e^{\lim_{x\to\infty} (\ln x) \frac{1}{x}} = e^0 = 1$$

($\frac{\ln x}{x} \to 0$, $x \to \infty$ růstová ř.

nebo l'Hosp.)

4. $$\lim_{n\to\infty} \left[\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} \right] = \lim_{n\to\infty} \left[\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right) \right] =$$

$$= \lim_{n \to \infty} \left[\frac{1}{1} - \frac{1}{n+1} \right] = 1 .$$

Př Symboly $o, O, \sim$

1) $\operatorname{arctg} x = O(1), x \to \infty$, tj. $\exists C \in \mathbb{R}$ $|f(x)| \le C|g(x)|$
$|f(x)| \le C$

$|\operatorname{arctg} x| \le \frac{\pi}{2}$

2) $x^2 e^{-x} = o(x^a)$, $a<0$ $\iff \lim\limits_{x\to\infty} \frac{x^{2-a}}{e^x}$ $\left| 2-a>0 \right|$
$x \to \infty$

$= \frac{(2-a)x^{1-a}}{e^x} \overset{\uparrow}{=} 0$
rnstovoužadou.

Nebo derivuji $(2-a)$-krát : $\frac{(2-a)(1-a)\cdots 1}{e^x} \overset{l'}{\underset{\frac{\infty}{\infty}}{=}}$

$\frac{0}{e^x} = 0.$

3) $\sqrt{x+\sqrt{x+\sqrt{x}}} \sim \sqrt{x}$, $x \to \infty$

$\lim\limits_{x\to\infty} \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}} = \lim\limits_{x\to\infty} \sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{x}} = \lim\limits_{x\to\infty} \sqrt{1+\sqrt{\frac{1}{x}+\underbrace{\frac{\sqrt{x}}{x^2}}_{\sqrt{\frac{x}{x^4}}}}} =$

$= \lim\limits_{x\to\infty} \sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}} = \lim\limits_{y\to 0} \sqrt{1+\sqrt{y+\sqrt{y^3}}} = 1.$