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Mathematics for Economists I
Problems 1

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You should be able to solve all given problems using high school mathematics, maybe with an exception – No. 13, where you need to know how to find roots of a cubic function.

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A little warm-up – simplification of expressions

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Simplify the given expressions for real variables a, b, c, w, x, y, z and determine the domain of definition, if applicable. (To save space, there are several problems separated by semicolons in every line.)

1. Expand: $(x + 3)^2$; $(y + 1)^3$; $(5 - z)^2$; $(-w - 4)^2$; $(x - 3)(x^2 + 2)$.

2. Factor out the highest possible term: $24y^2 + 6y - 30$; $9z^4 - 21z^2 + 33z^3$.

3. Simplify the expression: $(x^2)^5$; $(x^5)^2$; $x^{(2^5)}$; $x^{(5^2)}$; $(\sqrt{y})^6$; $\sqrt[3]{z^6}$; $\sqrt{\frac{x^3 x^7}{x^2}}$; $\sqrt[3]{w^2} w^2 \sqrt[3]{w}$.

4. Simplify the expression: $\dfrac{\frac{ab}{a^2}}{\frac{b^2 a}{ab}}$; $\dfrac{ac}{b^2} : \dfrac{c^2}{b}$; $\dfrac{3c}{10} - \dfrac{2a}{6} + \dfrac{b}{3}$.

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Equations and inequalities

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Find all real numbers x satisfying the given condition. Try to draw a picture to every problem.

5. $8 - 3x = 2$

6. $2x + 7 < 5$

7. $14 - 3x \geq 6$

8. $|x - 1| \leq 2$

9. $|5x - 3| \leq |x + 1|$

10. $||x - 4| - 3| = 2$

11. $x^2 - 6x - 7 \geq 0$

12. $15 > x^2 + 2x$

13. $x^3 + 10x^2 + 27x + 18 > 0$

14. $\dfrac{x+2}{x-6} < 0$

15. $\dfrac{x^2 - 6x + 5}{x - 3} \geq 0$

16. $\dfrac{x^2 + 2x - 8}{x^2 - 4x - 5} \leq 0$

17. $\sqrt{x} = 3$

18. $x^2 = 16$

19. $\sqrt[3]{x} = -2$

20. $x - 5\sqrt{x} + 4 = 0$

21. $x - 2\sqrt{x} - 3 = 0$

Applications

22. The mean exchange rate of the euro to the Czech koruna according to the exchange rate chart of the Czech National Bank as of September 4, 2018 was 1 EUR for 25.730 CZK, the exchange rate for the koruna and the Hungarian forint was 7.856 CZK for 100 HUF. How many forints should one euro cost (when converted via CZK)?

23. A product was first discounted by 30%, then increased by 30% from the new price. Is it now cheaper or more expensive than the original price? By what percentage of the original price is it? If it now costs 546 CZK, what was its original price?

Solutions: **1.** $x^2 + 6x + 9; y^3 + 3y^2 + 3y + 1; 25 - 10z + z^2; w^2 + 8w + 16; x^3 - 3x^2 + 2x - 6.$ **2.** $6(4y^2 + y - 5); 3z^2(3z^2 - 7 + 11z).$ **3.** $x^{10}; x^{10}; x^{32}; x^{25}; y^3$ (for $y \geq 0$); $z^2; x^4$ (for $x \neq 0$); $w^3.$ **4.** $\frac{1}{a}$ (for $a, b \neq 0$); $\frac{a}{bc}$ (for $b, c \neq 0$); $\frac{9c - 10a + 10b}{30}.$ **5.** 2. **6.** $(-\infty, -1).$ **7.** $(-\infty, \frac{8}{3}).$ **8.** $\langle -1, 3 \rangle.$ **9.** $\langle \frac{1}{3}, 1 \rangle.$ **10.** $\{-1, 3, 5, 9\}.$ **11.** $(-\infty, -1) \cup \langle 7, +\infty).$ **12.** $(-5, 3).$ **13.** $(-6, -3) \cup (-1, +\infty).$ **14.** $(-2, 6).$ **15.** $\langle 1, 3) \cup \langle 5, \infty).$ **16.** $\langle -4, -1) \cup \langle 2, 5).$ **17.** 9. **18.** $\pm 4.$ **19.** $-8.$ **20.** $\{1, 16\}.$ **21.** 9. **22.** 327,52 HUF for 1 EUR. **23.** The product is now cheaper by 9%. Its original price was 600 CZK.