Sample Midterm test – summer term 2024/25

In every problem, justify all steps properly.

1. (2 points) Find the limit

$$\lim_{n\to\infty} \frac{3n\sqrt{16n^2+7n+1}}{(2n+3)^2}$$

2. (2 points) Solve for x:

$$\log_2(x+2) + \log_2(3x-2) = 2 + \log_2(5x+2)$$

3. (2 points) Find the derivative of the function

$$f(x) = e^{(3x^2+5x)} + \frac{x-5}{x^2}$$

4. (4 points) A parabola is given as a graph of the function

$$f(x) = 2x^2 - 4x - 6.$$

- Find the equation of the tangent line to the parabola at the point $x_0 = 2$.
- Draw the parabola including its intercepts with the axes and vertex.
- Draw the previously found tangent line into the same picture, including its intercepts with the axes and its contact point with the parabola.

5. (10 points) Consider the function

$$f(x) = \frac{x^2+4x-5}{2x-6}$$

- Find its domain D_f.
- Find intercepts of f with axes.
- Find limits at all endpoints of D_f.
- Find the derivative of f.
- Find the intervals of monotony.
- Find the local, global extremes.
- Draw the graph.

Solutions:

1. 3

2. the only solution is 6, while $-\frac{2}{3}$ is a "false root", it does not lie in the domain of $\log_2(3x-2)$

3. $e^{3x^2+5x}(6x+5)+\frac{10-x}{x^3}$

4. tangent $y=4x-14$, contact point $[2,-6]$, roots $-1,3$, vertex $[1,-8]$

5. $D_f=\mathbb{R}-\{3\}$; $P_x:[-5,0],[1,0]$; $P_y=[0,\frac{5}{6}]$; $\lim\limits_{x\to-\infty} f(x)=-\infty$, $\lim\limits_{x\to3-} f(x)=-\infty$, $\lim\limits_{x\to3+} f(x)=+\infty$, $\lim\limits_{x\to\infty} f(x)=\infty$; $f'(x)=\frac{x^2-6x-7}{2(x-3)^2}$; increases in $(-\infty,-1)$ and in $(7,\infty)$, decreases in $(-1,3)$ and in $(3,7)$; loc. minimum $[7,9]$, loc. maximum $[-1,1]$.