

Mathematics for Economists I
Problems 4

Limit of a sequence

Find the limit of the given sequence:

1. $\lim_{n \rightarrow \infty} \frac{n+1}{2n+4}$
2. $\lim_{n \rightarrow \infty} \frac{n^2+n+1}{(n+3)^3+(n-1)^2}$
3. $\lim_{n \rightarrow \infty} \frac{4n^4+(n-3)^2}{n^2-(n+5)^3}$
4. $\lim_{n \rightarrow \infty} \frac{n^2+(n+1)^2}{(n+3)^2+(n+4)^2}$
5. $\lim_{n \rightarrow \infty} \left(\frac{2}{3}\right)^n - \frac{2n-1}{3n+7}$
6. $\lim_{n \rightarrow \infty} \frac{4n-3}{5n^2+6n-1} + (-2)^n$
7. $\lim_{n \rightarrow \infty} \sqrt{n}(\sqrt{n+1} - \sqrt{n})$
8. $\lim_{n \rightarrow \infty} n(\sqrt{n^2+1} - \sqrt{n^2-1})$
9. $\lim_{n \rightarrow \infty} 2 \cdot 5^n - 2 \cdot 4^{n+1}$
10. $\lim_{n \rightarrow \infty} \frac{3^{n+1}+5^n}{3^n-5^{n+1}}$
11. $\lim_{n \rightarrow \infty} \frac{3 \cdot 7^{n-1}+2^n}{5 \cdot 3^{n+1}-7^n}$
12. $\lim_{n \rightarrow \infty} \frac{\left(\frac{1}{3}\right)^n + \left(\frac{1}{2}\right)^n}{\left(\frac{1}{4}\right)^n - \left(\frac{1}{2}\right)^n}$
13. $\lim_{n \rightarrow \infty} \sqrt{n^2+n+1} - \sqrt{n^2-n-1}$
14. $\lim_{n \rightarrow \infty} \frac{\sqrt{n+4} \cdot \sqrt{4n+9} \cdot \sqrt{9n+16}}{(n+1)^2 - (n-1)^2}$
15. $\lim_{n \rightarrow \infty} \frac{\left(\frac{4}{3}\right)^{n+1} + \left(\frac{6}{5}\right)^{2n+1}}{\left(\frac{7}{5}\right)^{n+2} - \left(\frac{6}{5}\right)^{2n}}$
16. $\lim_{n \rightarrow \infty} \frac{\sqrt{n^3+3n^2} - \sqrt{n^3-3n^2}}{\sqrt{n}}$
17. $\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n+2} \cdot \sqrt[3]{n^2+2n+4}}{(n+1)^2 - (n-1)^2}$
18. $\lim_{n \rightarrow \infty} \frac{(n+2)^3 - (n-2)^3}{(2n+1)^2}$
19. $\lim_{n \rightarrow \infty} \frac{3 \cdot \left(\frac{1}{3}\right)^n + 5 \cdot \left(\frac{2}{5}\right)^n}{\left(\frac{1}{3}\right)^n + \left(\frac{2}{5}\right)^n}$

20. You sell your used bicycle for CZK 10 000; you want to make it cheaper gradually. Two options came to your mind: either to reduce the price by CZK 1 000 every month or to reduce the price by 20% every month. Analyze and compare them clearly. What would happen if time went „to infinity“?

Solutions: 1. $\frac{1}{2}$, 2. 0, 3. $-\infty$, 4. 1, 5. $-\frac{2}{3}$, 6. does not exist,
7. $\frac{1}{2}$, 8. 1, 9. $+\infty$, 10. $-\frac{1}{5}$, 11. $-\frac{3}{7}$, 12. -1 , 13. 1, 14. $+\infty$,
15. $-\frac{6}{5}$, 16. 3, 17. $\frac{1}{4}$, 18. 3, 19. 5, 20. left upon your own work.