Mathematics for Economists I
Problems 5

Limit of a sequence

Find the limit of the given sequence for $n \to \infty$:

1. $\frac{n+1}{2n+4}$

2. $\frac{n^2+n+1}{(n+3)^3+(n-1)^2}$

3. $\frac{4n^4+(n-3)^2}{n^2-(n+5)^3}$

4. $\frac{n^2+(n+1)^2}{(n+3)^2+(n+4)^2}$

5. $\left(\frac{2}{3}\right)^n - \frac{2n-1}{3n+7}$

6. $\frac{4n-3}{5n^2+6n-1} + (-2)^n$

7. $\sqrt{n}(\sqrt{n+1} - \sqrt{n})$

8. $n(\sqrt{n^2+1} - \sqrt{n^2-1})$

9. $2 \cdot 5^n - 2 \cdot 4^{n+1}$

10. $\frac{3^{n+1}+5^n}{3^n-5^{n+1}}$

11. $\frac{3 \cdot 7^{n-1}+2^n}{5 \cdot 3^{n+1}-7^n}$

12. $\frac{(\frac{1}{3})^n+(\frac{1}{2})^n}{(\frac{1}{4})^n-(\frac{1}{2})^n}$

13. $\sqrt{n^2+n+1} - \sqrt{n^2-n-1}$

14. $\frac{\sqrt{n+4} \cdot \sqrt{4n+9} \cdot \sqrt{9n+16}}{(n+1)^2-(n-1)^2}$

15. $\frac{\left(\frac{4}{3}\right)^{n+1}+\left(\frac{6}{5}\right)^{2n+1}}{\left(\frac{7}{5}\right)^{n+2}-\left(\frac{6}{5}\right)^{2n}}$

16. $\frac{\sqrt{n^3+3n^2}-\sqrt{n^3-3n^2}}{\sqrt{n}}$

17. $\frac{\sqrt[3]{n+2} \cdot \sqrt[3]{n^2+2n+4}}{(n+1)^2-(n-1)^2}$

18. $\frac{(n+2)^3-(n-2)^3}{(2n+1)^2}$

19. $\frac{3 \cdot \left(\frac{1}{3}\right)^n+5 \cdot \left(\frac{2}{5}\right)^n}{\left(\frac{1}{3}\right)^n+\left(\frac{2}{5}\right)^n}$

20. You are selling a bicycle for CZK 10,000; you want to make it cheaper gradually. Two options came to your mind: either to reduce the price by CZK 1,000 every month or to reduce the price by 20% every month. Analyze and compare them clearly. What would happen if time went „to infinity“?

Solutions: **1.** $\frac{1}{2}$, **2.** 0, **3.** $-\infty$, **4.** 1, **5.** $-2/3$, **6.** does not exist, **7.** $1/2$, **8.** 1, **9.** $+\infty$, **10.** $-\frac{1}{5}$, **11.** $-\frac{3}{7}$, **12.** -1, **13.** 1, **14.** $+\infty$, **15.** $-\frac{6}{5}$, **16.** 3, **17.** $\frac{1}{4}$, **18.** 3, **19.** 5, **20.** left upon your own work.