Mathematics for Economists I
Problems 12

Partial derivatives

Find partial derivatives of given functions with respect to all variables.

1. $f(x, y) = x^3y^2 + xy^3$
2. $f(x, y) = e^{xy^2}$
3. $f(x, y, z) = \ln(xz) + \frac{y}{z} - xy$

Stationary points of functions of several variables

Find all stationary points of the given function (consider $x, y, z \in \mathbb{R}$ if not mentioned otherwise).

4. $x^3 - 3x^2 + 4xy + y^2 + 8x$
5. $x^2 - 2xy + y^3 + 6y^2 + 3y$
6. $y^4 + 32x^2 - 32xy$
7. $x - y^2 - e^{x-2y}$
8. $x^3 + y^3 - 9xy + 15$
9. $\ln(x + 1) - xy^2; \quad x \in (-1, \infty), y \in \mathbb{R}$
10. $e^{x^2+(y+2)^2} + x^2$
11. $x^2 - 6x + y^2 + 2y + z^2 - 4z$
12. $xy - 2xz + 3yz + 7x - 15y + 3z$

Solutions:

1. $\partial_x f(x, y) = 3x^2y^2 + y^3$
$\partial_y f(x, y) = 2x^3y + 3xy^2$
2. $\partial_x f(x, y) = y^2e^{xy^2}$
$\partial_y f(x, y) = 2xye^{xy^2}$
3. $\partial_x f(x, y, z) = \frac{1}{x} - y$
$\partial_y f(x, y, z) = \frac{1}{z} - x$
$\partial_z f(x, y, z) = \frac{1}{z} + \frac{-y}{z^2}$ (all for $xz > 0$)
4. $[2/3, -4/3], [4, -8]$,
5. $[-3, -3], [-1/3, -1/3]$,
6. $[0, 0], [1, 2], [-1, -2]$,
7. $[2, 1]$,
8. $[0, 0], [3, 3]$,
9. $[0, 1], [0, -1]$,
10. $[0, -2]$,
11. $[3, -1, 2]$,
12. $[3, 1, 4]$.

Stationary points and extremes on polygons (substitution method)

For every given function f:
a) find all its stationary points in $\mathbb{R}^2$;
b) find its extrmes on the triangle A, B, C.

13. $f(x,y) = x^2 - 6xy - 3y^2 - 8x,\ A = [0,1], B = [0,-2], C = [3,-2]$.

14. $f(x,y) = -3x^2 + 6xy + y^2 - 6x + 6y,\ A = [-2,1], B = [1,1], C = [-2,-2]$.

15. $f(x,y) = 4x^2 - 2xy + y^2 - 6y,\ A = [0,2], B = [4,2], C = [0,6]$.

16. $f(x,y) = x^2 + 2xy + 4y^2 + 2x + 8y,\ A = [-2,0], B = [2,0], C = [-2,-4]$

17. $f(x,y) = x^2 + 4xy + 2y^2 + 2x + 4y,\ A = [1,2], B = [1,-2], C = [-1,-2]$.

Solutions:

	stat. point	min	max
13.	$(1,-1)$	$f(0,-2) = -12$	$f(3,-2) = 9$
14.	$(-1,0)$	$f(-2,1) = -5$	$f(-2,-2) = 16$
15.	$(1,4)$	$f(1,4) = -12$	$f(4,2) = 40$
16.	$(0,-1)$	$f(0,-1) = -4$	$f(-2,-4) = 48$
17.	$(-1,0)$	$f(1,-2) = -5$	$f(1,2) = 27$