VARIANTA E

1.

$$f(x) = \frac{18-2x^2}{x^2-4x-5}$$

DEFINIČNÝ OBOR

- menovateľ sa nesmie rovnať nule

$$x^2-4x-5 \neq 0$$

$$x_1+x_2=4 \quad \Rightarrow \quad x_1=-1$$
$$x_1 \cdot x_2 = -5 \qquad x_2=5$$

$$(x+1)(x-5) \neq 0 \Leftrightarrow x \neq \{-1, 5\}$$

$$\boxed{D_f = \mathbb{R} \setminus \{-1,5\}}$$

PRIESEČNÍKY S OSAMI

$P_y: x=0 \quad y = \frac{18-2\cdot 0^2}{0^2-4\,0-5} = -\frac{18}{5} = -3.6 \quad \boxed{[0, -\frac{18}{5}]}$

$P_x: y=0 \quad 0 = \frac{18-2x^2}{x^2-4x-5}$

$$0 = 18-2x^2$$
$$0 = 9-x^2$$
$$x^2 = 9$$
$$|x| = 3$$
$$x = \pm 3 \quad \boxed{[-3,0]\,;\,[3,0]}$$

INTERVALY, KDE JE FCIA KLADNÁ/ZÁPORNÁ

$$f(x) = \frac{18-2x^2}{x^2-4x-5} = \frac{-2(x-3)(x+3)}{(x-5)(x+1)} \qquad NB: \{-3,-1,3,5\}$$

	$(-\infty,-3)$	$(-3,-1)$	$(-1,3)$	$(3,5)$	$(5,\infty)$
$(x-3)$	−	−	−	+	+
$(x+3)$	−	+	+	+	+
$(x-5)$	−	−	−	−	+
$(x+1)$	−	−	+	+	+
$f(x)$	−	+	−	+	−

fcia je kladná na intervale

$$\boxed{(-3,-1) \cup (3,5)}$$

fcia je záporná na intervale

$$\boxed{(-\infty,-3) \cup (-1,3) \cup (5,\infty)}$$

2.

$$\lim_{n\to\infty} \frac{\left(\frac{1}{5}\right)^n + 4\left(\frac{1}{4}\right)^n}{\left(\frac{3}{5}\right)^n + 2\left(\frac{1}{2}\right)^{2n}} = \lim_{n\to\infty} \frac{\left(\frac{1}{5}\right)^n + 4\left(\frac{1}{4}\right)^n}{\left(\frac{3}{5}\right)^n + 2\left(\frac{1}{4}\right)^n} = \left| \begin{array}{l} \frac{1}{4} > \frac{1}{5} \\ \frac{1}{4} = \frac{9}{36} > \frac{2}{9} = \frac{8}{36} \end{array} \right| = \lim_{n\to\infty} \frac{\left(\frac{1}{4}\right)^n \left[\frac{\left(\frac{1}{5}\right)^n}{\left(\frac{1}{4}\right)^n} + 4\right]}{\left(\frac{1}{4}\right)^n \left[\frac{\left(\frac{3}{5}\right)^n}{\left(\frac{1}{4}\right)^n} + 2\right]} =$$

$$= \lim_{n\to\infty} \frac{\overbrace{\left(\frac{4}{5}\right)^n}^{\to 0} + 4}{\underbrace{\left(\frac{3}{5}\right)^n}_{\to 0} + 2} = \underline{\underline{2}}$$

③

$$f(x) = \frac{\sqrt{x^2+3x+7}}{4-3x}$$

$$4-3x \neq 0 \Rightarrow x \neq \frac{4}{3}$$

$$x^2+3x+7 \geq 0$$

$D = b^2 - 4 \cdot a \cdot c$

$= 3^2 - 4 \cdot 1 \cdot 7 = 9 - 28 = -19 < 0$

$\Rightarrow$ nemá reálne korene ; $a > 0 \Rightarrow \cup$

$\Rightarrow$ výraz je kladný pre všetky x

$$\boxed{D_f = \mathbb{R} \setminus \left\{ \frac{4}{3} \right\}}$$

$$\lim_{x \to \infty} \frac{\sqrt{x^2+3x+7}}{4-3x} = \lim_{x \to \infty} \frac{\sqrt{x^2\left(1+\frac{3}{x}+\frac{7}{x^2}\right)}}{x\left(\frac{4}{x}-3\right)} =$$

$$= \lim_{x \to \infty} \frac{|x| \cdot \sqrt{1+\underbrace{\frac{3}{x}}_{\to 0}+\underbrace{\frac{7}{x^2}}_{\to 0}}}{x\left(\underbrace{\frac{4}{x}}_{\to 0}-3\right)} = -\frac{1}{3}$$

poznámka:

$$\frac{|x|}{x} = \operatorname{sgn} x = \begin{cases} +1 & \text{ak } x > 0 \\ -1 & \text{ak } x < 0 \end{cases}$$

príklad:

$$\frac{|3|}{3} = \frac{3}{3} = 1 \qquad \frac{|-3|}{-3} = \frac{3}{-3} = -1$$

po vykrátení nám pred výrazom zostane znamienko z nekonečna

$$\lim_{x \to -\infty} \frac{|x| \cdot \sqrt{1+\frac{3}{x}+\frac{7}{x^2}}}{x\left(\frac{4}{x}-3\right)} = \lim_{x \to -\infty} -\frac{\sqrt{1+\underbrace{\frac{3}{x}}_{\to 0}+\underbrace{\frac{7}{x^2}}_{\to 0}}}{\underbrace{\frac{4}{x}}_{\to 0}-3} = +\frac{1}{3}$$

$$\lim_{x \to \frac{4}{3}^+} \frac{\sqrt{x^2+3x+7}}{4-3x} = \frac{+}{0^-} = -\infty$$

$$\lim_{x \to \frac{4}{3}^-} \frac{\sqrt{x^2+3x+7}}{4-3x} = \frac{+}{0^+} = +\infty$$

④

a) $f(x) = \ln\left(4x^2 + \frac{3}{x}\right)$

$$f'(x) = \frac{1}{4x^2+\frac{3}{x}} \cdot \left(8x + (-1) \cdot \frac{3}{x^2}\right) = \frac{8x - \frac{3}{x^2}}{\frac{4x^3+3}{x}} = \frac{\frac{8x^3-3}{x^2}}{\frac{4x^3+3}{x}} = \frac{8x^3-3}{(4x^3+3)x}$$

b) $g(x) = \dfrac{e^{1+5x}}{x^2+4x-1}$

$$g'(x) = \frac{e^{1+5x} \cdot 5 \cdot (x^2+4x-1) - e^{1+5x}(2x+4)}{(x^2+4x-1)^2} = \frac{e^{1+5x}\left(5x^2+20x-5-2x-4\right)}{(x^2+4x-1)^2}$$

$$= \frac{e^{1+5x}\left(5x^2+18x-9\right)}{(x^2+4x-1)^2}$$