BDU – Varianta D

1. $f(x) = \dfrac{\sqrt{-x^2+9x-18}}{3x-15}$

TABULKA:

- $D_f = ?$

$3x - 15 \neq 0$ $\qquad -x^2+9x-18 \geq 0$

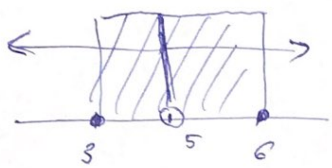
$x \neq 5$ $\qquad -(x-3)(x-6) \geq 0$

$x_0 = 3, 6$ (nulové body)

$(-\infty,3)$	$(3,6)$	$(6,+\infty)$	
+	−	−	$-(x-3)$
−	−	+	$x-6$
⊖	⊕	⊖	

$\to x \in \langle 3,6 \rangle$

Průnik $\langle 3,6 \rangle$ a $\mathbb{R} \setminus \{5\}$



$$\boxed{D_f = \langle 3,5) \cup (5,6\rangle}$$

Na intervalu $\langle 3,5)$ je funkce záporná, na $(5,6\rangle$ je funkce kladná.

Průsečíky: $P_y: x = 0$

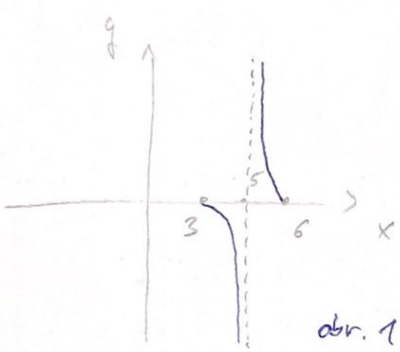
$f(0) = \dfrac{\sqrt{-18}}{-15} \to$ neexistuje

$P_x: y = 0$

$\sqrt{-x^2+9x-18} = 0$

$\sqrt{-(x-3)(x-6)} = 0$

$$\boxed{\begin{array}{l} P_{x_1} = [3,0] \\ P_{x_2} = [6,0] \end{array}}$$



obr. 1

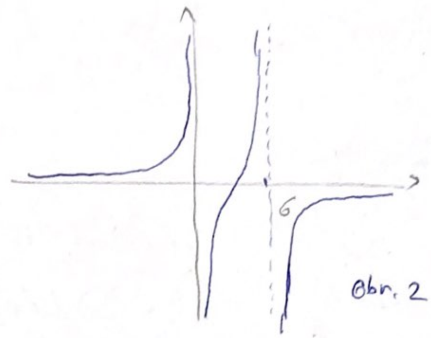
2. $\lim\limits_{n\to+\infty}\left(\sqrt{n^2+5n+7} - \sqrt{n^2-n+9}\right) \overset{(F3)}{=} \lim\limits_{n\to+\infty}\left(\sqrt{n^2+5n+7} - \sqrt{n^2-n+9}\right)\dfrac{\sqrt{n^2+5n+7}+\sqrt{n^2-n+9}}{\sqrt{n^2+5n+7}+\sqrt{n^2-n+9}}$

$= \lim\limits_{n\to+\infty}\dfrac{(n^2+5n+7)-(n^2-n+9)}{\sqrt{n^2+5n+7}+\sqrt{n^2-n+9}} = \lim\limits_{n\to+\infty}\dfrac{6n-2}{\sqrt{n^2+5n+7}+\sqrt{n^2-n+9}} \overset{(F1)}{=}$

$= \lim\limits_{n\to+\infty}\dfrac{n\left(6-\frac{2}{n}\right)}{n\left(\sqrt{1+\frac{5}{n}+\frac{7}{n^2}}+\sqrt{1-\frac{1}{n}+\frac{9}{n^2}}\right)} \overset{\text{VOAL}}{=} \dfrac{\lim\limits_{n\to+\infty}\left(6-\overset{\to 0}{\frac{2}{n}}\right)}{\lim\limits_{n\to+\infty}\left(\sqrt{1+\underset{\to 0}{\frac{5}{n}}+\underset{\to 0}{\frac{7}{n^2}}}+\sqrt{1-\underset{\to 0}{\frac{1}{n}}+\underset{\to 0}{\frac{9}{n^2}}}\right)}$

$= \dfrac{6}{2} = \underline{\underline{3}}$

(3.) $f(x) = \dfrac{16-4x}{x^2-6x} = \dfrac{16-4x}{x(x-6)}$



Obr. 2

$D_f = \mathbb{R} \setminus \{0, 6\}$ okrajové body: $0, 6, \pm\infty$

$$\lim_{x\to+\infty} \frac{16-4x}{x(x-6)} \overset{(F1)}{=} \lim_{x\to+\infty} \frac{\cancel{x}\left(-4+\frac{16}{x}\right)}{\cancel{x}(x-6)} = 0$$

$$\lim_{x\to-\infty} \frac{16-4x}{x(x-6)} \overset{(F1)}{=} \lim_{x\to+\infty} \frac{\cancel{x}\left(-4+\frac{16}{x}\right)}{\cancel{x}(x-6)} = 0$$

$$\lim_{x\to0^+} \frac{16-4x}{x(x-6)} = \frac{16}{0^+(0^+-6)} = -\infty$$

$$\lim_{x\to0^-} \frac{16-4x}{x(x-6)} = \frac{16}{0^-(0^--6)} = +\infty$$

súčin dvoch záporných čísel !

$$\lim_{x\to6^+} \frac{16-4x}{x(x-6)} = \frac{-8}{6\cdot0^+} = -\infty$$

$$\lim_{x\to6^-} \frac{16-4x}{x(x-6)} = \frac{-8}{6\cdot0^-} = +\infty$$

(4.) $f(x) = \ln(x^3-6x^2)$

$$f'(x) = \frac{1}{x^3-6x^2}\cdot(3x^2-12x) = \frac{3(x-4)}{x(x-6)}$$

$$g(x) = \frac{\sqrt{2+3x}}{2x^3+7}$$

$$g'(x) = \frac{\frac{3}{2\sqrt{2+3x}}(2x^3+7) - \sqrt{2+3x}\cdot6x^2}{(2x^3+7)^2} = \frac{3(2x^3+7) - 2(2+3x)6x^2}{2(2x^3+7)^2\sqrt{2+3x}} =$$

$$= \frac{6x^3+21-24x^2-36x^3}{2(2x^3+7)^2\sqrt{2+3x}} = -\frac{3(10x^3+8x^2-7)}{2(2x^3+7)^2\sqrt{2+3x}}$$