Bodovaný DÚ: Variantа (B)

1) $f(x) = \dfrac{x^2 - 8x + 12}{-x^2 + 6x + 7}$

D_f: Menovateľ $\neq 0$

$-x^2 + 6x + 7 \neq 0$

$x^2 - 6x - 7 \neq 0$

$(x-7)(x+1) \neq 0$

$x \neq 7$

$x \neq -1$

$\boxed{D_f = \mathbb{R} \setminus \{-1, 7\}}$

P_x: $y = 0 \Rightarrow 0 = \dfrac{x^2 - 8x + 12}{-x^2 + 6x + 7}$

$0 = x^2 - 8x + 12$

$0 = (x-6)(x-2)$

$x_1 = 6$

$x_2 = 2$

$\Rightarrow \boxed{P_{x_1} = [2, 0] \quad P_{x_2} = [6, 0]}$

P_y: $x = 0 \Rightarrow y = \dfrac{0^2 - 8 \cdot 0 + 12}{-0^2 + 6 \cdot 0 + 7}$

$y = \dfrac{12}{7}$

$\Rightarrow \boxed{P_y = [0, \tfrac{12}{7}]}$

Kladnosť, zápornosť:

Metóda nulových bodov:

NB: $-x^2 + 6x + 7 = 0 \iff x = -1 \vee x = 7$

$x^2 - 8x + 12 = 0 \iff x = 2 \vee x = 6$

NB: $-1, 2, 6, 7$

$\longrightarrow$ dosádzame do $f(x)$ čísla z intervalu a určujeme znam podľa znamienka podielu

$(-\infty, -1)$, $(-1, 2)$, $(2, 6)$, $(6, 7)$, $(7, \infty)$

$-1 \quad 2 \quad 6 \quad 7$

$\frac{+}{-}$ $-$ $\quad \frac{+}{+}$ $+$ $\quad \frac{-}{+}$ $-$ $\quad \frac{+}{+}$ $+$ $\quad \frac{+}{-}$ $-$

$\boxed{\begin{array}{l} \text{Funkcia je kladná pre } x \in (-1, 2) \cup (6, 7) \\ \text{Funkcia je záporná pre } x \in (-\infty, -1) \cup (2, 6) \cup (7, \infty) \end{array}}$

(2) $$\lim_{n\to\infty} \frac{\sqrt{n^3+2n^2}-\sqrt{n^3-2n^2}}{\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{n^3+2n^2}-\sqrt{n^3-2n^2}}{\sqrt{n}} \cdot \frac{\sqrt{n^3+2n^2}+\sqrt{n^3-2n^2}}{\sqrt{n^3+2n^2}+\sqrt{n^3-2n^2}}$$

rozšírime, aby sme použili $(a+b)(a-b) = a^2-b^2$

$$= \lim_{n\to\infty} \frac{n^3+2n^2-(n^3-2n^2)}{\sqrt{n}\left(\sqrt{n^3+2n^2}+\sqrt{n^3-2n^2}\right)} = \lim_{n\to\infty} \frac{n^3+2n^2-n^3+2n^2}{\sqrt{n}\cdot\left(\sqrt{n^3(1+\frac{2}{n})}+\sqrt{n^3(1-\frac{2}{n})}\right)}$$

$$= \lim_{n\to\infty} \frac{4n^2}{\sqrt{n}\left(\sqrt{n^3}\cdot\sqrt{1+\frac{2}{n}}+\sqrt{n^3}\cdot\sqrt{1-\frac{2}{n}}\right)} \quad \because \sqrt{a\cdot b} = \sqrt{a}\cdot\sqrt{b}$$

$$= \lim_{n\to\infty} \frac{4\cancel{n^2}}{\cancel{\sqrt{n}\cdot\sqrt{n^3}}\left(\sqrt{1+\frac{2}{n}}+\sqrt{1-\frac{2}{n}}\right)} \quad \because \sqrt{n}\cdot\sqrt{n^3} = n^{\frac{1}{2}}\cdot n^{\frac{3}{2}} = n^{\frac{1}{2}+\frac{3}{2}} = n^2$$

$$= \lim_{n\to\infty} \frac{4}{\sqrt{1+\underbrace{\tfrac{2}{n}}_{0}}+\sqrt{1-\underbrace{\tfrac{2}{n}}_{0}}} = \frac{4}{2} = \underline{\underline{2}} \quad \because \lim_{n\to\infty}\frac{2}{n} = 0$$

③ $f(x) = \dfrac{\sqrt{4x^2+x+1}}{x+3}$

$D_f: \; x+3 \neq 0 \quad \wedge \quad 4x^2+x+1 \geq 0$

$x \neq -3$

$D = 1^2 - 4\cdot 4\cdot 1 = -15 < 0$

↳ riešenie neexistuje → graf funkcie $g(x) = 4x^2+x+1$ nepretína os x

↳ je to parabola ($a=4$ → konvexná)

→ celá nad, podm. je splnená

$\to \boxed{D_f = \mathbb{R} \setminus \{-3\}}$

$$\lim_{x\to+\infty} \frac{\sqrt{4x^2+x+1}}{x+3} = \lim_{x\to+\infty} \frac{\sqrt{x^2\left(4+\frac{1}{x}+\frac{1}{x^2}\right)}}{x\left(1+\frac{3}{x}\right)} = \lim_{x\to+\infty} \frac{\cancel{|x|}\sqrt{4+\overset{\to 0}{\frac{1}{x}}+\overset{\to 0}{\frac{1}{x^2}}}}{\cancel{x}\left(1+\underset{\to 0}{\frac{3}{x}}\right)} =$$

$$|x| \xrightarrow{x\to+\infty} x$$

$$= \lim_{x\to+\infty} \frac{|x|}{x} \cdot \lim_{x\to+\infty} \frac{\sqrt{4+\frac{1}{x}+\frac{1}{x^2}}}{1+\frac{3}{x}} = 2$$

$\lim\limits_{x\to-\infty} \dfrac{\sqrt{4x^2+x+1}}{x+3} =$ ... ten istý postup ... $= \lim\limits_{x\to-\infty} \dfrac{|x|}{x} \cdot \lim\limits_{x\to-\infty} \dfrac{\sqrt{4+\frac{1}{x}+\frac{1}{x^2}}}{1+\frac{3}{x}}$

$|x| \xrightarrow{x\to-\infty} -x$ $\qquad = -2$

$$\lim_{x\to-3^-} \frac{\sqrt{4x^2+x+1}}{x+3} = \frac{\sqrt{32}}{0^-} = -\infty$$

$$\lim_{x\to-3^+} \frac{\sqrt{4x^2+x+1}}{x+3} = \frac{\sqrt{32}}{0^+} = +\infty$$

④

a) $f(x) = e^{2x^5-4x}$ ∵ derivácia zloženej fcie

$$f'(x) = e^{2x^5-4x} \cdot (10x^4 - 4)$$

b) $g(x) = \dfrac{\ln(x-3)}{x^2+5}$ ∵ derivácia podielu

$$g'(x) = \frac{\frac{1}{x-3}\cdot(x^2+5) - \ln(x-3)\cdot 2x}{(x^2+5)^2}$$

→ už tento tvar je dobrý

$$= \frac{\frac{x^2+5-2x\ln(x-3)(x-3)}{x-3}}{(x^2+5)^2}$$

$$= \frac{x^2+5-2x(x-3)\ln(x-3)}{(x-3)(x^2+5)^2}$$

∵ úpravy (ťažko povedať, či je zjednoduš