1. $\lim_{n\to\infty} \sqrt{n}\cdot\left(\sqrt{n^3+3n+1}-\sqrt{n^3-3n-1}\right)$

$= \lim_{n\to\infty} \sqrt{n}\cdot \frac{n^3+3n+1-(n^3-3n-1)}{\sqrt{n^3+3n+1}+\sqrt{n^3-3n-1}}$

$= \lim_{n\to\infty} \sqrt{n}\cdot \frac{6n+2}{n^{\frac{3}{2}}\cdot\left(\sqrt{1+\frac{3}{n^2}+\frac{1}{n^3}}+\sqrt{1-\frac{3}{n^2}-\frac{1}{n}}\right)}$

$= \lim_{n\to\infty} \frac{\sqrt{n}}{n^{\frac{3}{2}}}\cdot(6n+2)\cdot \underbrace{\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{3}{n^2}+\frac{1}{n^3}}+\sqrt{1-\frac{3}{n^2}-\frac{1}{n}}}}$

$= \frac{1}{\sqrt{1+0+0}+\sqrt{1+0+0}} = \frac{1}{2}$

$= \frac{1}{2}\cdot\lim_{n\to\infty} n^{\frac{1}{2}-\frac{3}{2}}\cdot(6n+2)$

$= \frac{1}{2}\cdot\lim_{n\to\infty}\frac{6n+2}{n} = \frac{1}{2}\lim_{n\to\infty}\frac{\cancel{n}\left(6+\frac{2}{n}\right)}{\cancel{n}} = \frac{1}{2}\cdot 6 = \underline{\underline{3}}$

2. $f(x)=\frac{1}{2}x^2+4x+6$, $x_0=-8$

ROVNICE TEČNY: $y-y_0=f'(x_0)\cdot(x-x_0)$

$f'(x)=x+4$

$f'(-8)=-4$

TEČNÝ BOD: $[-8;6]$

$y_0=f(-8)=32-32+6=6$

TEČNA: $\boxed{y-6=(-4)\cdot(x+8)}$

$\Rightarrow \quad y=-4x-26$

průsečíky s osami: $y=0 \iff \frac{1}{2}x^2+4x+6=0 \quad |\cdot 2$

$x^2+8x+12=0$

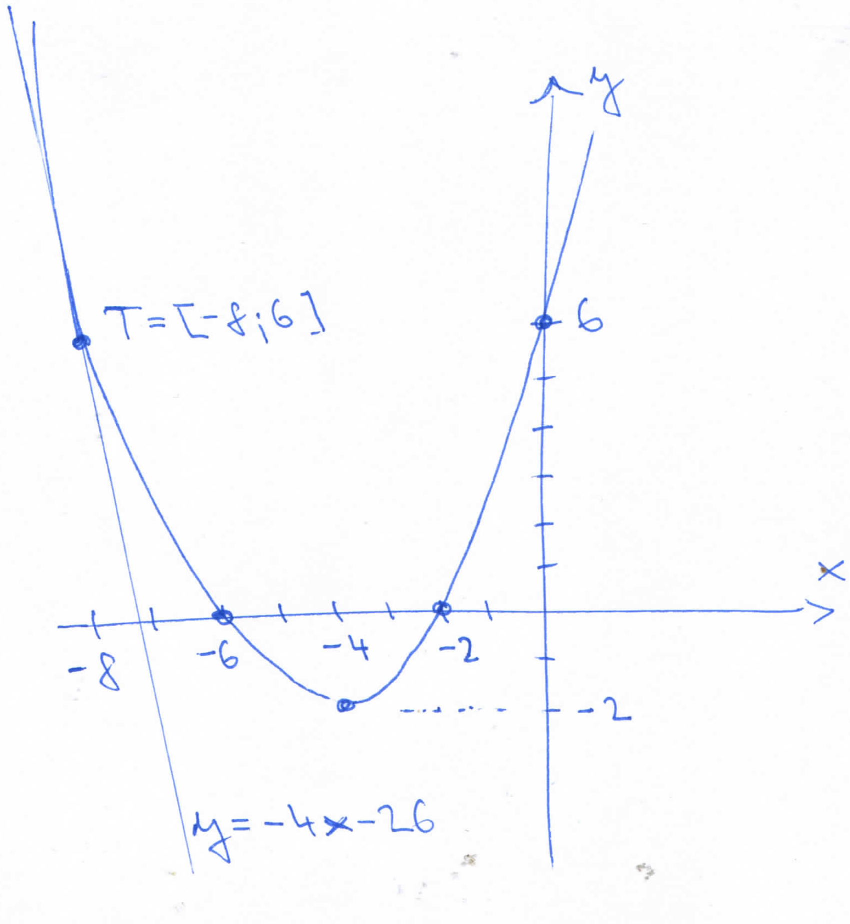
$(x+6)(x+2)=0$

$\underline{x=-6} \vee \underline{x=-2}$

x	-2	-6	0	-4
$f(x)$	0	0	6	-2

$f'(x)=0 \iff x=-4$

$f(-4)=8-16+6=-2$



$f(x) = 2(x-1)\cdot e^{2-x}$ $\quad D_f = \mathbb{R}$

$f'(x) = 2\cdot e^{2-x} + 2(x-1)\cdot e^{2-x}\cdot(-1) = e^{2-x}\cdot(2-2x+2)$

$f'(x) = e^{2-x}\cdot(4-2x)$

$f'(x) = 0 \iff 4-2x = 0$

$\underline{\underline{x=2}}$

$f'>0 \quad 2 \quad f'<0$

lok. maximum : $[2;2]$

$f(2) = 2\cdot(2-1)\cdot e^0 = 2$

$f''(x) = -e^{2-x}\cdot(4-2x) + e^{2-x}\cdot(-2) = e^{2-x}\cdot(2x-6)$

$f''(x) = 0 \iff 2x-6 = 0$

$\underline{x=3}$

$f''<0 \quad 3 \quad f''>0$

$\forall x \in (3,\infty): f''(x) > 0 \Rightarrow$ funkce je KONVEXNÍ

$\forall x \in (-\infty,3): f''(x) < 0 \Rightarrow$ funkce je KONKÁVNÍ

inflexní bod: $[3; \frac{4}{e}]$

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to\infty} \frac{2(x-1)}{e^{x-2}} \stackrel{L.P.}{=} \lim\limits_{x\to\infty} \frac{2}{e^{x-2}} = \frac{2}{+\infty} = \underline{\underline{0}}$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \frac{2(x-1)}{e^{x-2}} = \frac{-\infty}{0_+} = \underline{\underline{-\infty}}$

šikmá asymptota: $k = \lim\limits_{x\to\pm\infty} \frac{f(x)}{x} = \lim\limits_{x\to-\infty} \frac{2(x-1)e^{2-x}}{x} =$

$= 2\cdot\lim\limits_{x\to-\infty}\left(1-\frac{1}{x}\right)\cdot e^{2-x} = 2\cdot e^{+\infty} = +\infty$

$\Rightarrow$ nemá šikmou asymptotu v $-\infty$

v $+\infty$ má vodorovnou asymptotu $y=0$.

$\forall x \in (1, \infty): f(x) > 0$
$\forall x \in (-\infty, 1): f(x) < 0$

x	1	0	2	3
y	0	$-2e^2$	2	$\frac{4}{e}$

průsečíky s osami: $y = 0 \iff x = 1$
$x = 0 \iff y = -2e^2$

Obor hodnot: $H_f = (-\infty, 2\rangle$