1.

$$\lim_{n\to\infty} \frac{\left(\frac{1}{3}\right)^{n+1}+\left(\frac{1}{2}\right)^{2n}}{\left(\frac{1}{3}\right)^{n-1}+\left(\frac{1}{2}\right)^{2n}} = \lim_{n\to\infty} \frac{\cancel{\left(\frac{1}{3}\right)^{n}}\cdot\left(\left(\frac{1}{3}\right)^{1}+\left(\frac{3}{4}\right)^{n}\right)}{\cancel{\left(\frac{1}{3}\right)^{n}}\cdot\left(\left(\frac{1}{3}\right)^{-1}+\left(\frac{3}{4}\right)^{n}\right)}$$

$$= \frac{\frac{1}{3}+0}{\left(\frac{1}{3}\right)^{-1}+0} = \frac{\frac{1}{3}}{3} = \underline{\underline{\frac{1}{9}}}$$

$$\left(\frac{1}{3}\right)^{n+1} = \left(\frac{1}{3}\right)^{n}\cdot\left(\frac{1}{3}\right)^{1}$$

$$\left(\frac{1}{2}\right)^{2n} = \left(\left(\frac{1}{2}\right)^{2}\right)^{n} = \left(\frac{1}{4}\right)^{n}$$

$$\boxed{q^{n} \xrightarrow[n\to\infty]{} 0 \text{ pro } q\in(-1,1)}$$

2. $f(x) = -x^2+11x-28$, $x_0 = 2$

$f'(x) = -2x+11$

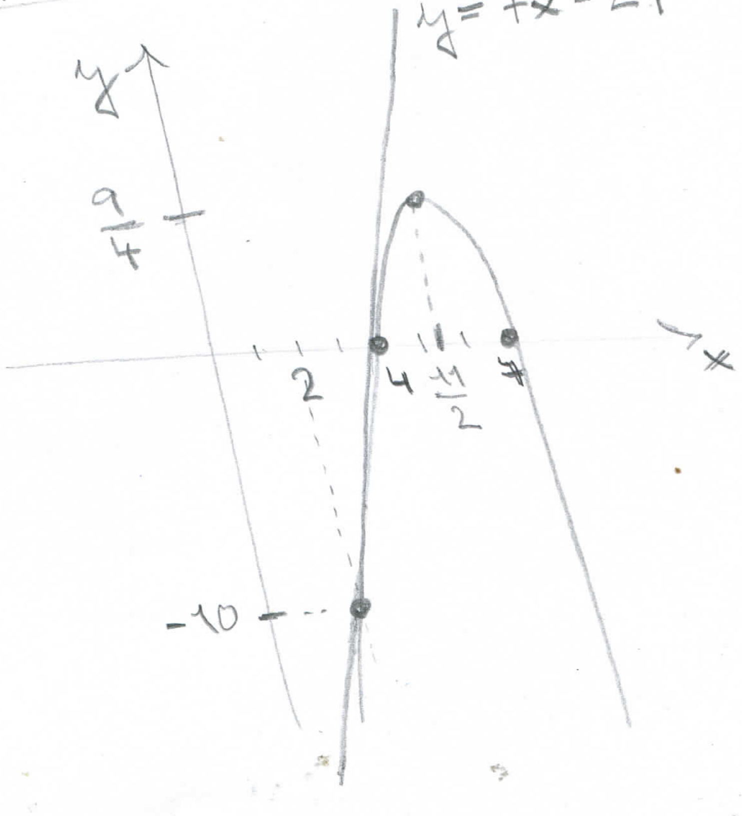
$f'(2) = -2\cdot 2+11 = 7$

$y_0 = f(2) = -4+22-28 = -10$

ROVNICE TEČNY: $\boxed{y-y_0 = f'(x_0)\cdot(x-x_0)}$

$y+10 = 7\cdot(x-2)$

$\boxed{y = 7x-24}$



$f(x) = -x^2+11x-28$

$= (-1)(x^2-11x+28)$

$= (-1)(x-4)(x-7)$

x	4	7	$\frac{11}{2}$
y	0	0	$\frac{9}{4}$

VRCHOL: $f\left(\frac{11}{2}\right) = (-1)\left(\frac{3}{2}\cdot\left(-\frac{3}{2}\right)\right)$

$f\left(\frac{11}{2}\right) = \frac{9}{4}$

$$f(x) = \frac{x^2+x-2}{4-2x}$$

x	1	-2	0	4
$f(x)$	0	0	$-\frac{1}{2}$	$-\frac{9}{2}$

$D_f = \mathbb{R} \setminus \{2\}$

průsečíky s osami: $y=0 \iff x^2+x-2=0$

$$(x+2)(x-1)=0$$

$$\underline{x=-2} \vee \underline{x=1}$$

$P_y = [0; -\frac{1}{2}]$

$P_{x_1} = [1; 0]$

$P_{x_2} = [-2; 0]$

$$f'(x) = \frac{(2x+1)(4-2x) - (x^2+x-2)\cdot(-2)}{(4-2x)^2} = \frac{6x+4-4x^2+2x^2+2x-4}{(4-2x)^2}$$

$$= \frac{8x-2x^2}{(4-2x)^2} = \frac{2x\cdot(4-x)}{(4-2x)^2}$$

	$\searrow$	0	$\nearrow$	2	$\nearrow$	4	$\searrow$
	$f'<0$		$f'>0$		$f'>0$		$f'<0$

$f'(x)=0 \iff \underline{x=0} \vee \underline{x=4}$

lok. minimum: $[0; -\frac{1}{2}]$

lok. maximum: $[4; -\frac{9}{2}]$ $\qquad f(4) = \frac{16+4-2}{4-8} = -\frac{9}{2}$

$$f''(x) = \left(\frac{8x-2x^2}{(4-2x)^2}\right)' = \frac{(8-4x)(4-2x)^2 - (8x-2x^2)\cdot 2(4-2x)(-2)}{(4-2x)^4}$$

$$= \frac{\cancel{(4-2x)}\cdot\left((8-4x)(4-2x)+4\cdot(8x-2x^2)\right)}{(4-2x)^{\cancel{4}\,3}}$$

$$= \frac{32-16x-16x+8x^2+32x-8x^2}{(4-2x)^3} = \frac{32}{(4-2x)^3}$$

$\forall x \in (2, +\infty): f''(x) < 0 \Rightarrow$ funkce je konkávní

$\forall x \in (-\infty, 2): f''(x) > 0 \Rightarrow$ funkce je konvexní

$\forall x \in D_f: f''(x) \neq 0 \Rightarrow f$ nemá inflexní bod

$\cup$		$\cap$
$f''>0$	2	$f''<0$

Šikmá asymptota:

$$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{\frac{x^2+x-2}{4-2x}}{x} = \lim_{x \to \pm\infty} \frac{x^2+x-2}{4x-2x^2}$$

$$\overset{L.P.}{=} \lim_{x \to \pm\infty} \frac{2x+1}{4-4x} \overset{L.P.}{=} \lim_{x \to \pm\infty} \frac{2}{-4} = \underline{\underline{-\frac{1}{2}}}$$

$$q = \lim_{x \to \pm\infty} (f(x) - kx) = \lim \left(\frac{x^2+x-2}{4-2x} + \frac{1}{2}x \right)$$

$$= \frac{1}{2} \cdot \lim \left(\frac{x^2+x-2}{2-x} + x \right) = \frac{1}{2} \lim \frac{x^2+x-2+2x-x^2}{2-x}$$

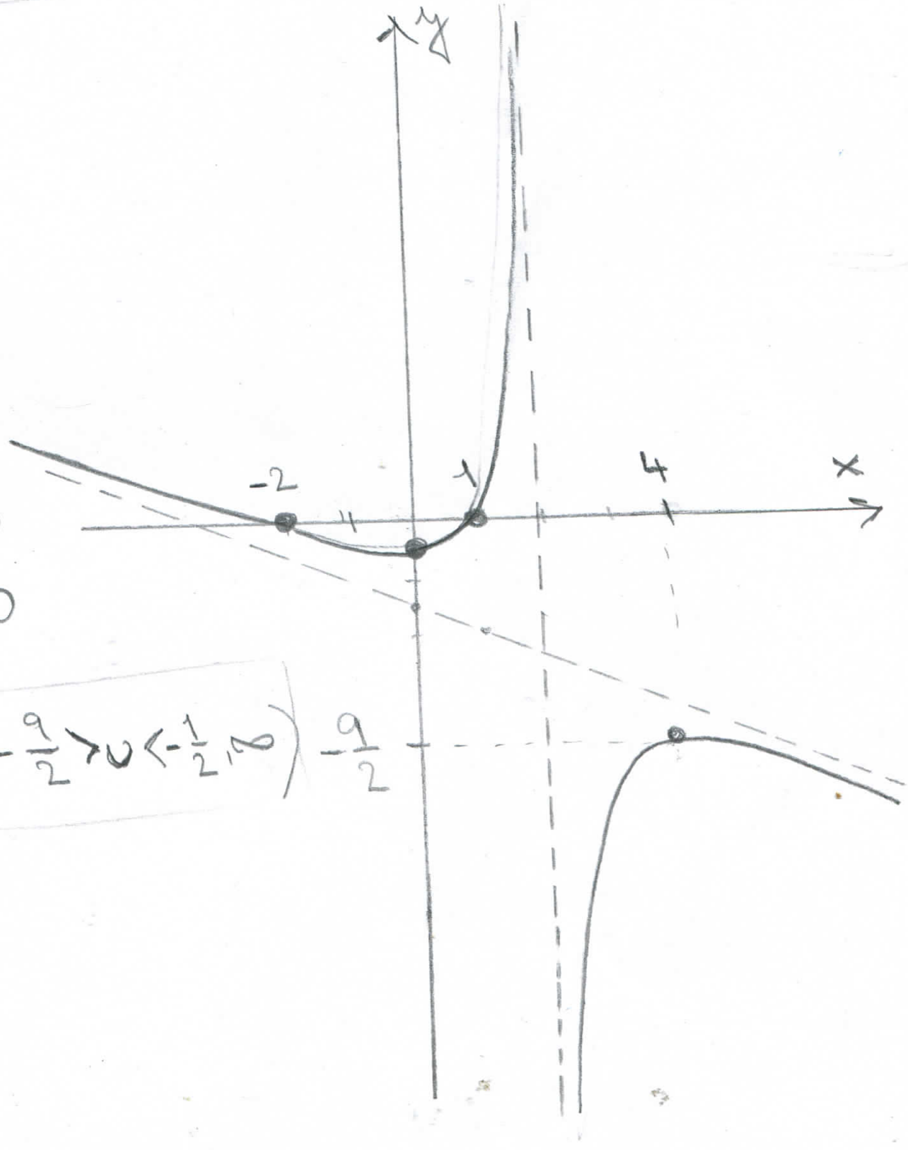
$$= \frac{1}{2} \lim_{x \to \pm\infty} \frac{3x-2}{2-x} \overset{L.P.}{=} \frac{1}{2} \cdot \lim \frac{3}{(-1)} = \underline{\underline{-\frac{3}{2}}}$$

$\Rightarrow$ přímka $\boxed{y = -\frac{1}{2}x - \frac{3}{2}}$ je šikmá asymptota v $\pm\infty$

$$f(x) = \frac{(x+2)(x-1)}{2(2-x)}$$

	$(-\infty,-2)$	$(-2,1)$	$(1,2)$	$(2,\infty)$
$x+2$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$+$	$+$
$2-x$	$+$	$+$	$+$	$-$
	$+$	$-$	$+$	$-$

$-\infty \quad -2 \quad 1 \quad 2 \quad \infty$



$\forall x \in (-\infty,-2) \cup (1,2) : f(x) > 0$

$\forall x \in (-2,1) \cup (2,\infty) : f(x) < 0$

Obor hodnot: $\boxed{H_f = (-\infty, -\frac{9}{2}\rangle \cup \langle -\frac{1}{2}, \infty)}$