

KOHERENCE VaR a CVaR (5)

MONOTONIE:

necht $Y \geq X$ s.j. Pak $[Y-a]^+ \geq [X-a]^+$ s.j.

necht $a_Y = \arg \min_{a \in \mathbb{R}} \left\{ a + \frac{1}{1-\alpha} E[Y-a]^+ \right\}$ pro $\forall a \Rightarrow E[Y-a]^+ \geq E[X-a]^+$

$$\begin{aligned} \text{pak: } CVaR_{\alpha}(Y) &= a_Y + \frac{1}{1-\alpha} E[Y-a_Y]^+ \\ &\geq a_Y + \frac{1}{1-\alpha} E[X-a_Y]^+ \\ &\geq \min_a \left\{ a + \frac{1}{1-\alpha} E[X-a]^+ \right\} \\ &= CVaR_{\alpha}(X) \quad \checkmark \end{aligned}$$

SUBADITIVITA:

UVĚDOME SI, ŽE PLATÍ: $[x+y]^+ = \max(x+y, 0) \leq$

necht a_x, a_y jsou taková, že: $\max(0, x) + \max(0, y) = [x]^+ + [y]^+$

$$CVaR_{\alpha}(X) = a_x + \frac{1}{1-\alpha} E[X-a_x]^+$$

$$CVaR_{\alpha}(Y) = a_y + \frac{1}{1-\alpha} E[Y-a_y]^+$$

Pak:

$$\begin{aligned} d CVaR_{\alpha}(X) + (1-d) CVaR_{\alpha}(Y) &= d a_x + (1-d) a_y + \frac{d}{1-\alpha} E[X-a_x]^+ + \frac{(1-d)}{1-\alpha} E[Y-a_y]^+ \\ &\geq d a_x + (1-d) a_y + \frac{1}{1-\alpha} E \left[d(X-a_x) + (1-d)(Y-a_y) \right]^+ \\ &= \underbrace{d a_x + (1-d) a_y}_b + \frac{1}{1-\alpha} E \left[(dX + (1-d)Y) - (d a_x + (1-d) a_y) \right]^+ \\ &\geq \min_b \left[b + \frac{1}{1-\alpha} E[dX + (1-d)Y - b]^+ \right] = CVaR_{\alpha}(dX + (1-d)Y) \quad \checkmark \end{aligned}$$