Convex Optimization 2025/26

Practical session # 4

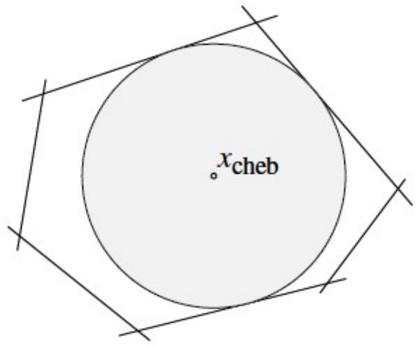
October 23, 2025

1. Perform the simplex method for the problem:

$$\begin{aligned} \text{maximize } & f(x,y,z) = 2x + 3y + 4z \\ \text{subject to } & 3x + 2y + z \le 10 \\ & 2x + 5y + 3z \le 15 \\ & x, y, z \ge 0 \end{aligned}$$

   Discuss, which entering/leaving variable is the best choice in the first pivot step.

2. Let $P = \{x \in \mathbf{R}^n \mid a_i^T x \le b_i, i = 1, \ldots, m\}$ be a polytope. The *Chebyshev center* of P is the center of the largest Euclidean ball $B = \{x_c + u \mid \|u\| \le r\}$ that lies in P. Model the problem of finding the Chebyshev center as a Linear Program.



Recall the definition of the following norms on $\mathbf{R}^n$:

- $\|x\|_\infty = \max(|x_1|, |x_2|, \ldots, |x_n|)$
- $\|x\|_1 = |x_1| + |x_2| + \ldots + |x_n|$

3. Discuss the n-dimensional crosspolytope $\{x \in \mathbf{R}^n \mid \|x\|_1 \le 1\}$. What are its vertices? Describe it as the intersection of 2^n halfspaces.

4. The norm approximation problem for a system of linear equation $Ax = b$ is the problem of minimizing $\|Ax - b\|$, for a given norm $\|\cdot\|$. Show that both

   (a) minimize $\|Ax - b\|_\infty$

   (b) minimize $\|Ax - b\|_1$

   are equivalent to an LP. Note that in (b) we are looking for an LP that is of polynomial size.