Convex Optimization 2025/26

Homework # 2
October 30, 2025

Instructions

- Please, submit your homeworks to kompatscher@karlin.mff.cuni.cz. The subject of your email should start with `[Convex Optimization]`.
- The written solutions are expected to be submitted in a single `.pdf` file and include your name. The use of LaTeX is encouraged - if you use handwriting, make sure it's legible. Additionally attach any Python code you used.
- There will be 4 homework assignments, on each of which you need to score at least 26 out of 40 points to obtain the credit (zápočet) for the course.
- Please, send your submissions no later than November 13, 15:40.

Exercise 1 (10 points) Solve the following linear program using the simplex method:

$$\begin{aligned} \text{minimize } & f(x_1, x_2, x_3) = -2x - 3y + z \\ \text{subject to } & x - y + z \geq 0 \\ & 3x + y \leq 1 \\ & x, y, z \geq 0 \end{aligned}$$

Exercise 2 (10 points) Find a solution formula for minimizing a linear function over an ellipsoid, i.e.

$$\begin{aligned} \text{minimize } & c^T x \\ \text{subject to } & (x - x_c)^T A (x - x_c) \leq 1, \end{aligned}$$

for given $A \in S^n_{++}$, $x_c \in \mathbf{R}^n$, and $c \neq 0$.
(Hint: it might help to reduce it to the problem of minimizing an affine function over the unit ball by a suitable coordinate transformation).

Exercise 3 (10 points) Let $A \in \mathbf{R}^{m \times n}, b \in \mathbf{R}^m, c \in \mathbf{R}^n, d \in \mathbf{R}$ such that $d > \|c\|_1$. Consider the problem

$$\begin{aligned} \text{minimize } & f_0(x) = \frac{\|Ax - b\|_1}{c^T x + d}, \\ \text{subject to } & \|x\|_\infty \leq 1. \end{aligned}$$

Here $\mathrm{dom}(f_0) = \{x \mid c^T x + d > 0\}$. Note that $d > \|c\|_1$ and $\|x\|_\infty \leq 1$ implies that $x \in \mathrm{dom}(f_0)$.

- Show that f_0 is quasi-convex.
- Show that the problem is equivalent to

$$\begin{aligned} \text{minimize } & g_0(y, t) = \|Ay - bt\|_1 \\ \text{subject to } & \|y\|_\infty \leq t, \\ & c^T y + dt = 1. \end{aligned}$$

(Hint: recall the reduction of linear-fractional problems to LPs)

Exercise 4 (10 points)

You are an anthropologist specialized on food. During your studies of Middle Earth you observed that some hobbits in the north cook with butter, while groups in the south prefer oil. On a map, you collect some data-points on towns who mainly use butter $B = \{b_1, b_2, \ldots, b_n\} \subseteq \mathbf{R}^2$ versus oil $O = \{o_1, o_2, \ldots, o_m\} \subseteq \mathbf{R}^2$. You would like to find a 'butter-oil-equator', i.e. a line $L = \{(x, y) \in \mathbf{R}^2 \mid y = ax + b\}$, for some $a, b \in R$ that best separates B and O. Note that an optimal such line should maximize the 'gap' d between L and both sets. Describe a linear program that is equivalent to finding such a line L.