Convex Optimization 2025/26

Homework # 1
October 16, 2025

Instructions

- Please, submit your homeworks to kompatscher@karlin.mff.cuni.cz. The subject of your email should start with `[Convex Optimization]`.
- The written solutions are expected to be submitted in a single `.pdf` file and include your name. The use of LaTeX is encouraged - if you use handwriting, make sure it's legible. Additionally attach any Python code you used.
- There will be 4 homework assignments, on each of which you need to score at least 26 out of 40 points to obtain the credit (zápočet) for the course.
- Please, send your submissions no later than October 30, 15:40.

Exercise 1 (5 points)
Which of the following sets are convex? Which of them are convex cones?

- $\{(a_0, \ldots, a_{n-1}) \in \mathbf{R}^n \mid a_0 + a_1 x + \cdots a_{n-1}x^{n-1} \geq 0 \text{ for all } x \in \mathbf{R}\}$ (non-negative polynomials)
- $\{A \in \mathbf{R}^{2\times 2} \mid \det(A) \geq 1\}$
- $\{x \in \mathbf{R}^n \mid \|x\|_2 \leq \|x - y\|_2 \text{ for all } y \in S\}$, where $S \subseteq \mathbf{R}^n$. (points closer to the origin than to S)

Exercise 2 (10 points) For which values of $\alpha \in \mathbf{R}$ is $f(x, y) = x^\alpha y^{1-\alpha}$ with $\operatorname{dom}(f) = \mathbf{R}^2_{++}$ convex/concave or neither? Is there some $\alpha \in \mathbf{R}$ for which f is strictly convex/concave?

Exercise 3 (10 points)
Consider the problem "minimize $f_0(x) = \frac{1}{2}x^T P x + q^T x + s$", where $P \in S_n$, $q \in \mathbf{R}^n$ and $s \in \mathbf{R}$. For which parameters P, q, s is f_0 (strictly) convex? In this case, discuss the optimal solutions. When are there no optimal solutions? When is there a unique one? In the latter case, compute the optimal solution and optimal value.

Exercise 4 (15 points) You are playing your favorite urban planning simulator, and want to find the optimal location to place a new hospital. The map of your city is a perfect square $B = [0, 100] \times [0, 100]$, from which you can pick a location $x \in B$ completely freely. Your goal is to find x such that its maximal distance (with respect to some norm $\|\cdot\|$) from a list of settlements $u_1, u_2, \ldots, u_m \in B$ is minimal.

1. Show that this is a convex optimization problem.
2. For the Euclidean norm ($\|\cdot\|_2$), find an equivalent problem that has quadratic objective and constraint functions (Hint: use the epigraph trick from Practical 2)
3. Using CVXPY, solve the problem for $u_1 = (8, 2), u_2 = (13, 60), u_3 = (15, 3), u_4 = (0, 60), u_5 = (12, 50)$.