NMAI076 - Algebra 2 - spring semester 2024/25

Homework 1

Deadline 20.3.2025, 14:00

Exercise 1. (10 points) Let $G = \{f\colon \mathbb{R} \to \mathbb{R}\colon f(x) = ax + b,\ a, b \in \mathbb{R}, a \neq 0\}$.

(1.1) Prove that G is a group with respect to the composition of functions.

(1.2) Prove that $N = \{f\colon \mathbb{R} \to \mathbb{R}\colon f(x) = x + b,\ b \in \mathbb{R}\}$ is a normal subgroup of G.

(1.3) Describe the quotient G/N.

Exercise 2. (10 points) Show that, up to isomorphism, there is exactly one group of order 15 (Hint: try to use a similar argument as in the lecture, when we classified the groups of order 6. You can use Cauchy's theorem, which states that for every prime $p \mid |G|$, G contains an element of order p).

Exercise 3. (10 points) Consider the dihedral group D_8 (the group of symmetries of the square, or equivalently $D_8 = \langle (1234), (14)(23) \rangle \leq S_4$).

(2.1) Determine the order of D_8 and the order of each of its elements.

(2.2) Determine (up to isomorphism) all homomorphic images of D_8.