NMAI 076 - Algebra 2 - spring semester 2022/23

Homework 1

Deadline 27.3.2023, 10:40

Exercise 1. (10 points) Let $G = \{f\colon \mathbb{R} \to \mathbb{R}\colon f(x) = ax + b,\ a, b \in \mathbb{R}, a \neq 0\}$.

(1.1) Prove that G is a group with respect to the composition of functions.

(1.2) Prove that $N = \{f\colon \mathbb{R} \to \mathbb{R}\colon f(x) = x + b,\ b \in \mathbb{R}\}$ is a normal subgroup of G.

(1.3) Describe the quotient G/N.

Exercise 2. (10 points) Consider the dihedral group D_4 (group of symmetries of the square).

(2.1) Determine the order of D_4 and the order of each of its elements.

(2.2) Determine up to isomorphism all homomorphic images of D_4.

Exercise 3. (10 points) Prove or disprove the following isomorphisms of rings.

(3.1) $\mathbb{C}[x]/(x^2 + x) \cong \mathbb{C}^2$.

(3.2) $2\mathbb{Z} \cong 3\mathbb{Z}$.