Linearization of certain non-trivial equations in oligomorphic clones

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CSPs and non-trivial equations

Constraint satisfaction problems

Let $\mathbb{A} = (A, R_1, \ldots, R_n)$ be a relational structure.

CSP($\mathbb{A}$)

INPUT: A primitive positive sentence

$$\phi = \exists x_1 \ldots, x_n R_{i_1}(\ldots) \wedge \cdots \wedge R_{i_j}(\ldots)$$

QUESTION: $\mathbb{A} \models \phi$?

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Conjecture (Feder, Vardi '98; Bulatov, Jeavons, Krokhin '02)

Let $\mathbb{A}$ be finite and Pol($\mathbb{A}$) be idempotent. Then either

1. There is a clone homomorphism ξ : Pol($\mathbb{A}$) $\to$ **1** (and CSP($\mathbb{A}$) is NP-complete)
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$\rightarrow$ in 2: study of non-trivial equations.

Non-trivial equations

Let $\mathcal{C}$ be a finite idempotent clone. Then TFAE:

1. $\mathcal{C}$ has no clone homomorphism to **1**
2. $\mathcal{C}$ has a Taylor operation
3. $\mathcal{C}$ has a weak near unanimity operation
   $w(y, x, \ldots, x) = w(x, y, x, \ldots, x) = \ldots = w(x, x, \ldots, y)$
4. $\mathcal{C}$ has a Siggers operation
   $s(x, y, x, z, y, z) = s(y, x, z, x, z, y)$
5. $\mathcal{C}$ has a cyclic operation
   $c(x_1, x_2, \ldots, x_n) = c(x_2, \ldots, x_n, x_1)$

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2-5 are examples of linear non-trivial equations: no nesting

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2. or Pol($\mathbb{A}$) satisfies a non-trivial linear equation and CSP($\mathbb{A}$) is in P.

Oligomorphic clones

The dichotomy conjecture for infinite CSPs

Old conjecture (Bodirsky, Pinsker)

Let $\mathbb{A}$ be a reduct of a finitely bounded homogeneous structure and $\mathbb{A}^c$ its model-complete core. Then either

1. There is a *uniformly continuous* clone homomorphism $\xi : \mathrm{Pol}(\mathbb{A}^c, a_1, \ldots, a_n) \to \mathbf{1}$ (and CSP($\mathbb{A}$) is NP-complete)
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Main question: Are the conjectures equivalent?

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Theorem (Barto, Pinsker '16)
$\mathcal{C}$... oligomorphic clone and model-complete core. Then either

- Some stabilizer $(\mathcal{C}, a_1, \dots, a_n) \to \mathbf{1}$ uniformly continuous or
- $\mathcal{C}$ contains a pseudo-Siggers operation s:

$$e_1 \circ s(x, y, x, z, y, z) = e_2 \circ s(y, x, z, x, z, y), \qquad e_1, e_2 \in \mathcal{C}.$$

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Potential approach
Is $e_1 \circ s(x, y, x, z, y, z) = e_2 \circ s(y, x, z, x, z, y)$ equivalent to a set of linear non-trivial equations?

Linearization with 🐦

Example: the clone of injective functions

For oligomorphic clones: non-trivial equations $\not\to$ Taylor operations

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Let $\mathcal{O}^{inj}$ be the clone generated by all injective operations $\mathbb{N}^n \to \mathbb{N}$.

Let $f(x, y) : \mathbb{N}^2 \to \mathbb{N}$ be a bijection, $f \in \mathcal{O}^{inj}$. Then $e : f(x, y) \to f(y, x)$ is a bijection, $e \in \mathcal{O}^{inj}$.

$\mathcal{O}^{inj}$ satisfies the non-trivial equation $f(y, x) = e \circ f(x, y)$.

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But, by injectivity $\mathcal{O}^{inj}$ contains no Taylor operation.

$\to$ we need more than one operation!

Pigeonhole principle 🐦 🕳

Lemma

Let $k > 2$ and $g_1(x, y), \ldots, g_{2k-1}(x, y) \in \mathcal{C}$. Assume that for every tuple $I = (i_1 < \cdots < i_k)$, there is an $f_I(x_1, \ldots, x_k) \in \mathcal{C}$, such that $\forall n$:

$$f_I(x, \ldots, x, \underset{\substack{\uparrow \\ n}}{y}, x, \ldots, x) = g_{i_n}(x, y).$$

Then this set of linear equations is non-trivial.

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Proof

Assume there is a clone homomorphism $\xi : \mathcal{C} \to \mathbf{1}$. For the binary functions $g_i(x,y)$, there are only two possible images $\pi_1^2(x,y)$ and $\pi_2^2(x,y)$.

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But then $\xi(f_I(x_1, \dots, x_k))$ cannot be a projection! ⚡ $\square$

Examples of CSP classifications

Successful CSP classifications for reducts of finitely bounded homogeneous structures:

- $(\mathbb{N}, =)$ (Equality CSPs; Bodirsky, Kára '06)
- $(\mathbb{Q}, <)$ (Temporal CSPs; Bodirsky, Kára '08)
- the random graph (Graph-SAT problems; Bodirsky, Pinsker '11)
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Theorem (BKOPP '16)

If $\mathbb{A}$ is a reduct of one of the above then either

- $\mathrm{Pol}(\mathbb{A}^c, a_1, \ldots, a_n) \to \mathbf{1}$ and CSP($\mathbb{A}$) is NP-complete
- or Pol($\mathbb{A}$) satisfies a set of non-trivial linear equations  and CSP($\mathbb{A}$) is in P

More linearization

Theorem (pseudo-nu operations)

Let $\mathbb{D}$ be a finitely bounded homogeneous structure, and let f be a strong polymorphism of $\mathbb{D}$ with

$$e(x) = e_1 \circ f(y, x \ldots, x) = e_2 \circ f(x, y, x \ldots, x) = \ldots = e_n \circ f(x, \ldots, x, y).$$

Then, f induces non-trivial linear equations. 🐦 🕳

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Theorem (totally symmetric operations)

Let $\mathbb{A}$ be a reduct of a finitely bounded homogeneous structure $\mathbb{D}$, k big enough and let $f(x_1, \dots, x_k) \in \mathrm{Pol}(\mathbb{A})$ be totally symmetric modulo outer embeddings of $\mathbb{D}$: $\forall \rho \in \mathrm{Sym}(k)$:

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Note: assumptions on the structural side!

The two conjectures are equivalent

The bad news

The bad news (BKOPP ’16)

For $\mathbb{B}$, the countable atomless Boolean algebra (extended by $\neq$):

- Pol($\mathbb{B}$) satisfies the equation $e_1 \circ f(x, y) = e_2 \circ f(y, x)$ and
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Here $\mathrm{Pol}(\mathbb{B})$ is oligomorphic, but $\mathbb{B}$ is not reduct of a finitely bounded homogeneous structure:

$\mathrm{Aut}(\mathbb{B})$ has double exponential orbit growth.

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Corollary: The two conjectures are equivalent!

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2. Understand better the relation between equations in $\mathrm{Pol}(\mathbb{A})$ and orbit growth of $\mathrm{Aut}(\mathbb{A})$.
3. When does $\xi\colon \mathrm{Pol}(\mathbb{A}) \to \mathbf{1}$ h1-clone homomorphism imply that there is also a uniformly continuous $\xi'\colon \mathrm{Pol}(\mathbb{A}) \to \mathbf{1}$?

Reference

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Equations in oligomorphic clones and the Constraint Satisfaction Problem for ω-categorical structures

arXiv:1612.07551

Thank you!