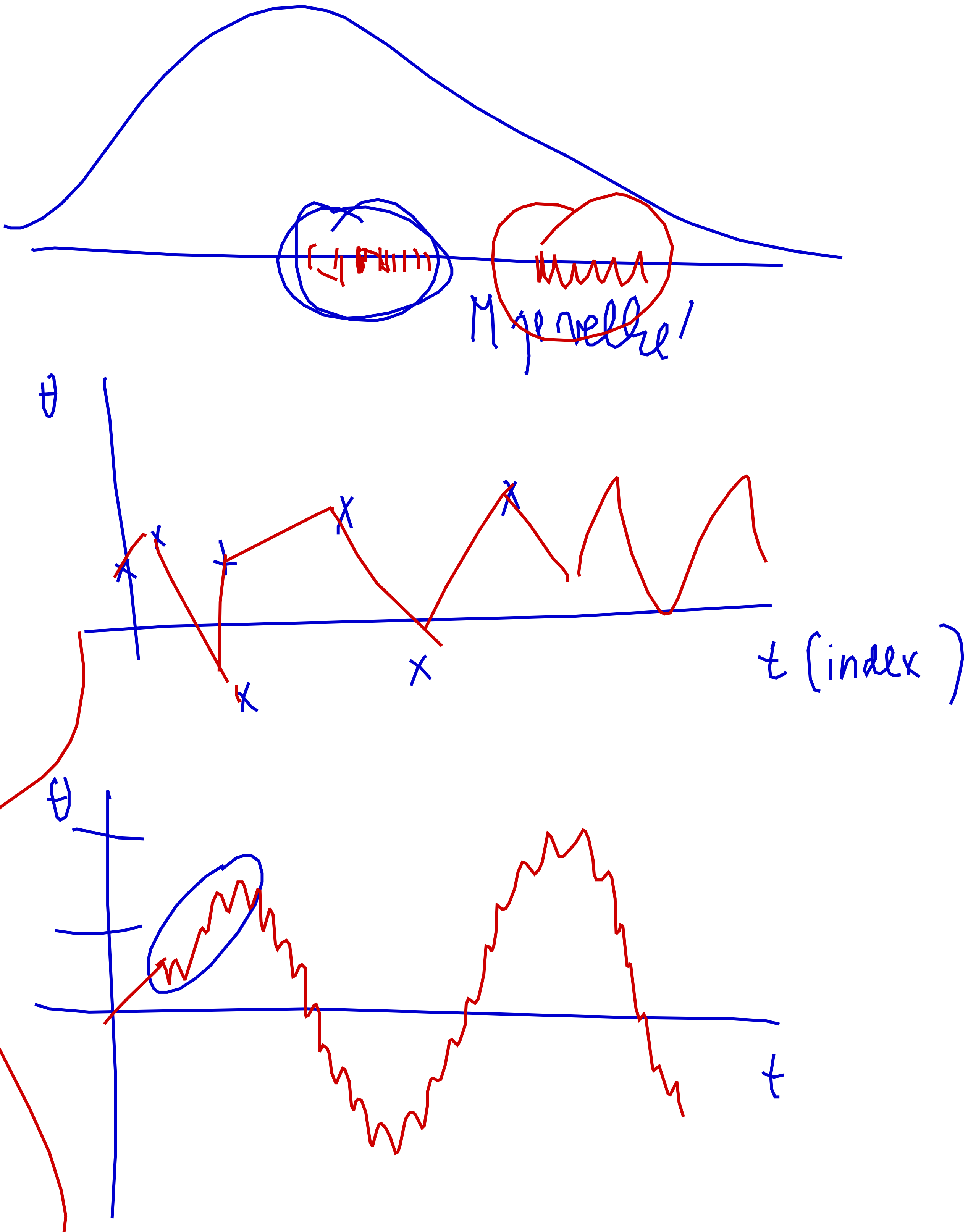


chi generat t

$$f(\theta) = f(\theta_1, \theta_2, \ldots, \theta_k) =$$

vždy
$$= f(\theta_1 | \theta_2, \ldots, \theta_k)\, f(\theta_2 | \theta_3, \ldots, \theta_k) \ldots$$
$$\ldots f(\theta_{k-1} | \theta_k)\, f(\theta_k)$$

Gibbs: chci umět generovat z
$$f(\theta_i | \theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_k)$$
$$= f(\theta_i | \theta_{-i})$$

---

gibbsův algoritmus

1. θ^0 (počáteční stav) $\sim f_0$ (libovolné)

2. $\theta^m \longrightarrow \theta^{m+1}$ ↓ algoritmus

θ^{m+1} generuju postupně po složkách

(i) $\theta_1^{(m+1)} \sim f(\theta_1 | \theta_2^{(m)}, \ldots, \theta_k^{(m)})$

(ii) $\theta_2^{(m+1)} \sim f(\theta_2 | \theta_1^{(m+1)}, \theta_3^{(m)}, \ldots, \theta_k^{(m)})$

(iii) $\theta_3^{(m+1)} \sim f(\theta_3 | \theta_1^{(m+1)}, \theta_2^{(m+1)}, \theta_4^{(m)}, \ldots, \theta_k^{(m)})$

$\vdots$

(k) $\theta_k^{(m+1)} \sim f\left(\theta_k \mid \theta_1^{(m+1)}, \theta_2^{(m+1)}, \ldots, \theta_{k-1}^{(m+1)}\right)$

---

přechodová hustota ?

$$p(\theta, \psi) = f(\psi_1 \mid \theta_2, \ldots, \theta_k)\, f(\psi_2 \mid \psi_1, \theta_3, \ldots, \theta_k) \cdots$$

$$\cdots f(\psi_{k-1} \mid \psi_1, \ldots, \psi_{k-2}, \theta_k)$$

$$\cdot f(\psi_k \mid \psi_1, \ldots, \psi_{k-1})$$

$(\equiv$ "$p(\psi \mid \theta)$"$)$

$\psi = (\psi_1, \psi_2, \ldots, \psi_k)$

→ přechod. jádro

$$P(\theta, T) = \int_T p(\theta, \psi)\, d\lambda(\psi)$$

Věta 5.1 Důkaz pro $\underline{k=2}$:

$\theta = (\theta_1, \theta_2)$

$$p(\theta, \psi) = f(\psi_1 \mid \theta_2) \cdot f(\psi_2 \mid \psi_1)$$

$\equiv$ přechod. hustota

? $fP(T) = f(T) \quad \forall T \in \mathcal{T}$

$$f(T) \overset{def}{=} \int_T f(\psi)\, d\lambda(\psi)$$

$$fP(T) \overset{def}{=} \int_{\Theta} P(\theta, T)\, f(\theta)\, d\lambda(\theta) =$$

$$= \int_{\Theta} \int_T p(\theta, \psi)\, d\lambda(\psi)\, \underline{f(\theta)}\, d\lambda(\theta) =$$

$$= \int_{\Theta_2} \int_{\Theta_1} \left(\int_T f(\psi_1 | \theta_2)\, f(\psi_2 | \psi_1)\, d\lambda(\psi) \right) \underline{f(\theta_1, \theta_2)}\, d\lambda(\theta_1)\, d\lambda(\theta_2) =$$

$$\overset{\text{Fub.}}{=} \int_T \int_{\Theta_2} \underbrace{\int_{\Theta_1} f(\theta_1, \theta_2)\, d\lambda(\theta_1)}_{f(\theta_2)}\, f(\psi_1 | \theta_2)\, d\lambda(\theta_2) \cdot f(\psi_2 | \psi_1)\, d\lambda(\psi) =$$

$$= \int_T \underbrace{\int_{\Theta_2} \underbrace{f(\theta_2) \cdot f(\psi_1 | \theta_2)}_{f(\psi_1, \theta_2)}\, d\lambda(\theta_2)} \cdot f(\psi_2 | \psi_1)\, d\lambda(\psi) =$$

$f(\psi_1)$

$$= \int_T \underbrace{f(\psi_1) \cdot f(\psi_2 | \psi_1)}_{f(\psi_1, \psi_2)} \, d\lambda(\psi) = \int_T f(\psi) \, d\lambda(\psi) = f(T)$$

---

algoritmus $\Rightarrow$ splňuje stacionaritu

$\Uparrow$

OBECNĚ reversibilita

$$\theta^{(m)} \xrightarrow[\text{gibbs odpředu}]{} \psi^{\text{mezitím}} \xrightarrow[\text{gibbs odzadu}]{} \theta^{(m+1)}$$

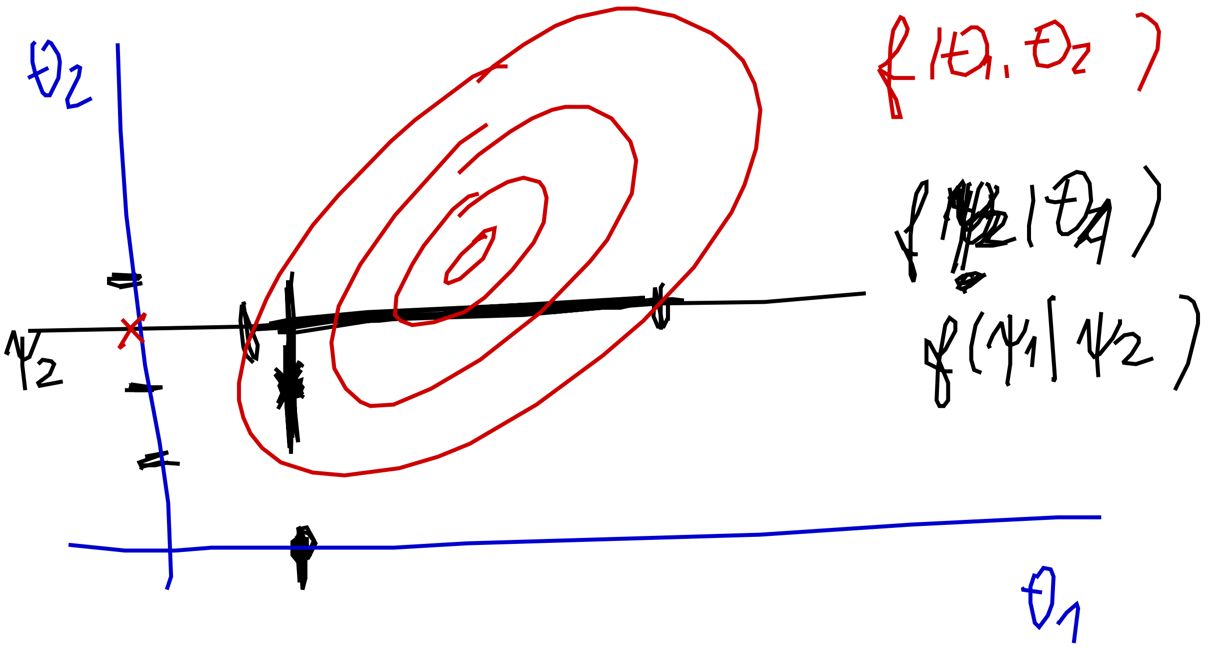
$$\theta^m \rightarrow \underbrace{\theta^{m+1}}$$

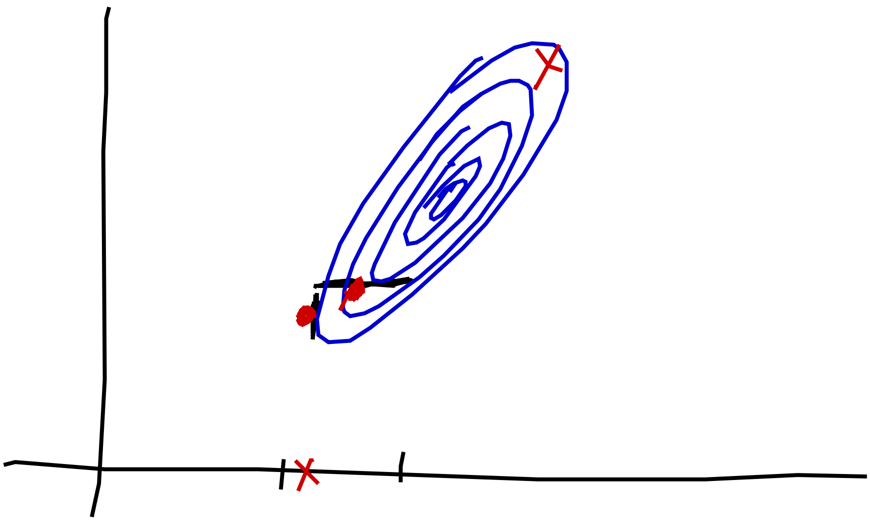
změní se pouze 1 podvektor
vybraný náhodně s pravděpodobn.
$p_1, \dots, p_k$

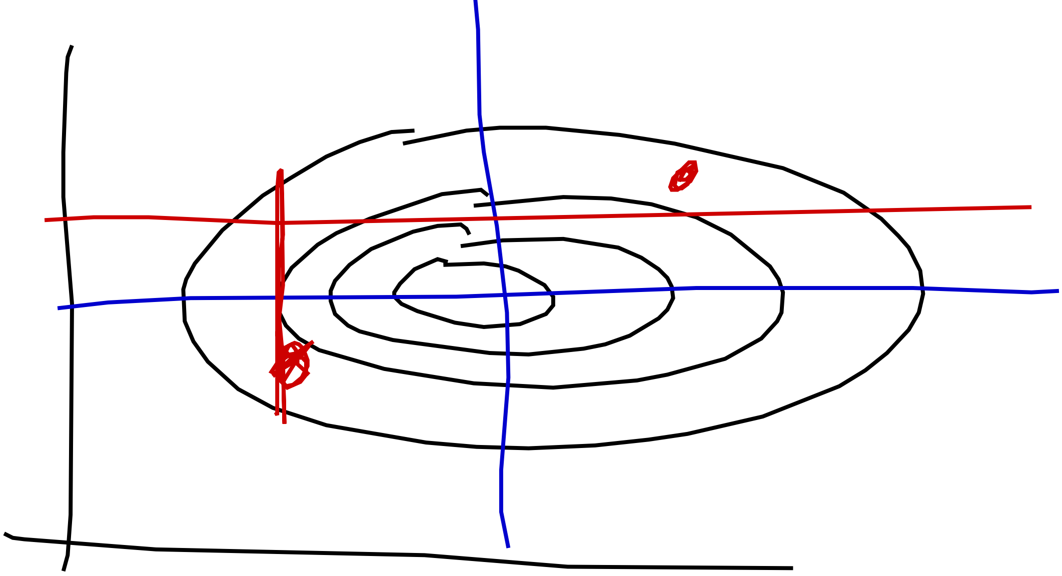
$f(\theta) = f(\theta_1, \dots, \theta_k)$

MOŽNOST: $\theta_1, \dots, \theta_k \in \mathbb{R}$

→ výpočet plně podmíněná jednorozměrná
(typicky vede k velké autokorelovanosti)







Bayes model

$p(y|\theta)$, $p(\theta)$ typicky $p(\theta_1) \cdot p(\theta_2) \dots p(\theta_k)$

$\to \underbrace{p(\theta|y)} \propto \underbrace{p(y|\theta)}_{\text{vyrobí závislost mezi } \theta_1, \dots, \theta_k} p(\theta)$

Typicky pro Gibbse

plné podmíněné rozklad (rozklad) odpovídá rozkladu apriorního rozdělení!

hierar. model

$$f(\theta) = p(\theta|y) \propto f_1(uzel_1 | \overset{\text{rodiče 1}}{\text{nadním}}) \cdot$$

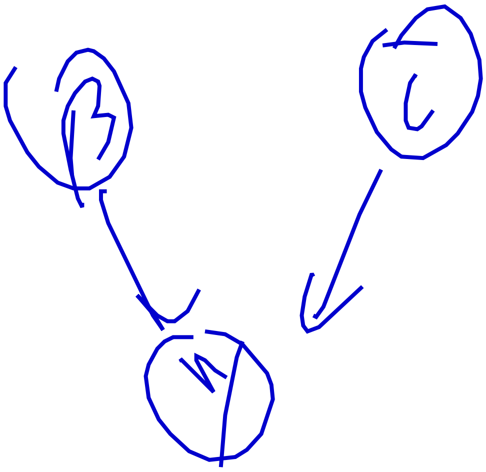
$$f_2(uzel_2 | \overset{\text{rodiče 2}}{\text{nadním}}) \ldots$$

...

$$f(\theta_i | \theta_{-i}) = \frac{f(\theta)}{\cancel{[illegible]}}$$

const vůči θ_i

$$\propto f(\theta) \propto \text{viz výše}$$



$$p(\beta,\tau|y) \propto p(y|\beta,\tau)\, p(\beta)\, p(\tau)$$

$$p(\beta|\underbrace{\ldots}_{\tau, y}) \propto \underbrace{p(\beta)}_{\propto 1} \cdot \underbrace{p(y|\beta,\tau)}_{\sim N} \sim N(\ldots)$$

$$p(\tau \mid \ldots) \propto \underbrace{p(\tau)}_{\sim Ga(0,0)} \cdot \underbrace{p(y \mid \beta, \tau)}_{\sim N}$$

β, y

$$p(\tau) \propto \frac{1}{\tau}$$

$$p(\beta_i \mid \ldots) \propto p(\beta) \cdot p(y \mid \beta, \tau)$$

Gibbs: $(\underset{\theta_1}{\beta}, \underset{\theta_2}{\tau})$ $\quad p(\beta \mid \ldots)$

$$p(\tau \mid \ldots)$$