$Y_i = \text{hmohm}_i + \varepsilon_i \quad , \quad \varepsilon_i \sim N(0, \sigma^2)$

$EY_i = \text{hmohm}.$

$\beta_1 = \text{hm}.\,1 \, , \quad \beta_2 = \text{hm}.\,2$

$$X = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ \hline 0 & 1 \\ 0 & \vdots \\ 0 & 1 \\ \hline 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{pmatrix}$$

$$p(\beta, \tau \mid y) = p(\beta \mid \tau, y) \cdot p(\tau \mid y)$$

$\beta \mid \tau, y \sim N$

$\tau \mid y \sim Ga$

$$E(\beta \mid Y) = \int \beta \, p(\beta, \tau \mid y) \, d(\beta, \tau)$$

$$= \int \beta \underbrace{\left(\int p(\beta, \tau \mid y) \, d\tau \right)}_{p(\beta \mid y)} d\beta$$

$$E(\beta_j \mid Y) = b_j \quad (= \text{odhad MNČ})$$

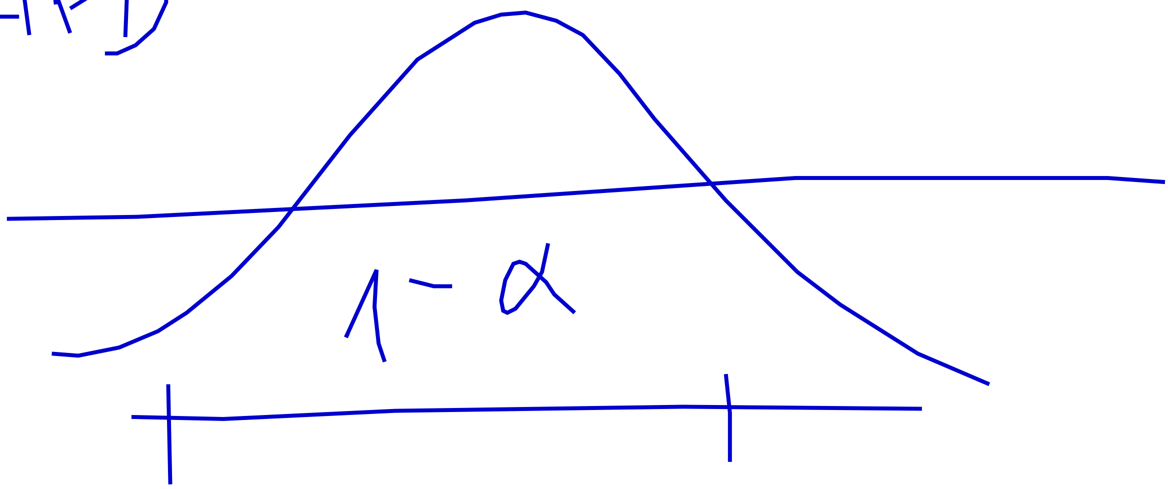
Credible interval:

- ET :

$$P(\beta_j \in I_\alpha \mid Y) = 1-\alpha$$

$$\beta_j \quad b_j \pm t_{n-k}\left(1-\frac{\alpha}{2}\right)\sqrt{\ldots}$$

• HPD



? věrohodná množina pro β ?

$$P\left(\frac{1}{2}(\beta-b)^T(\quad)(\beta-b) < F^{-1}(1-\alpha|y)\right) = 1-\alpha$$

$\{\beta : \frac{1}{2}(\quad) < F^{-1}(1-\alpha|y)$

= elipsoid

$$E(\tau) = \frac{c_0}{d_0}, \quad var(\tau) = \frac{c_0}{d_0^2}$$

$$Ga(c_0, d_0) \longrightarrow \frac{1}{\tau}$$

$$c_0 \to 0, \; d_0 \to 0$$

$n = 50 \quad Y_1, \ldots, Y_n \quad \bar{y} = 59{,}6$

$s = 11{,}1$

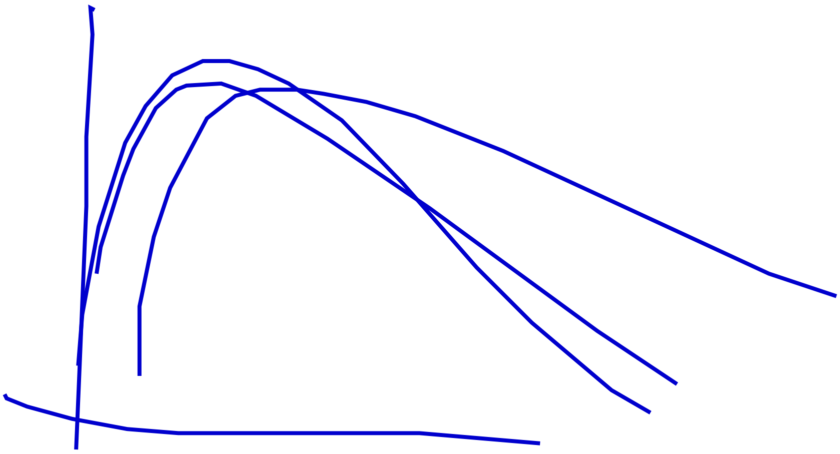
$Y_i \overset{iid}{\sim} N(\mu, \sigma^2), \quad \tau = \sigma^{-2}$

$p(\mu, \tau) = p(\mu) \cdot p(\tau)$

$p(\mu) \propto 1 \quad , \quad \tau \sim Ga(g, h)$

$E(\tau) = \frac{g}{h} \quad var(\tau) = \frac{g}{h^2}$

$P(\mu > 55 \mid y)$



$p(\sigma \mid y) = ?$

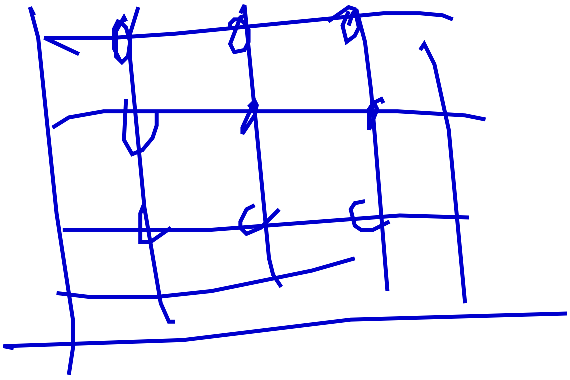
$p(\tau \mid y) = Ga$ věta o transf.

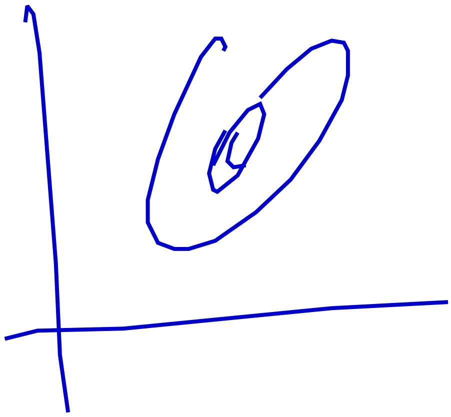
$\sigma = \sqrt{\frac{1}{\tau}} \longrightarrow$

$\tau = \frac{1}{\sigma^2}$

$$p(\sigma|y) = p(\tau(\sigma)|y)\cdot|\tau'(\sigma)|$$

$$f : \mathbb{R}^2 \to \mathbb{R}$$





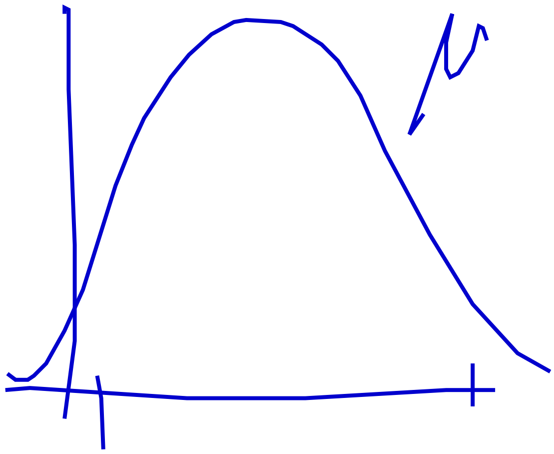
VI'M: $p(\mu,\sigma|y) \sim N\text{-}Gam(u)$

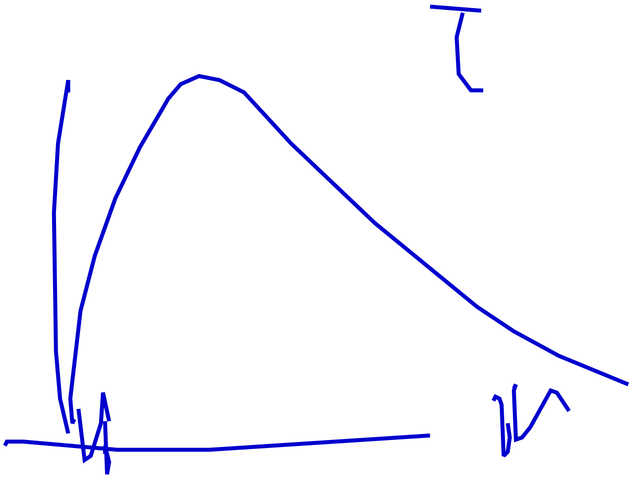
$$p(\mu,\sigma|y) =$$

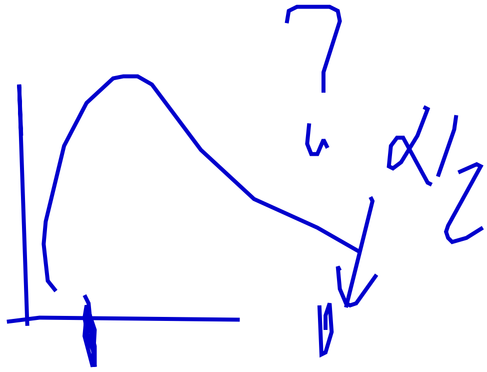
$$= p(\mu,\sigma|y) =$$

$$= p(\mu|\sigma,y)\cdot p(\sigma|y) =$$

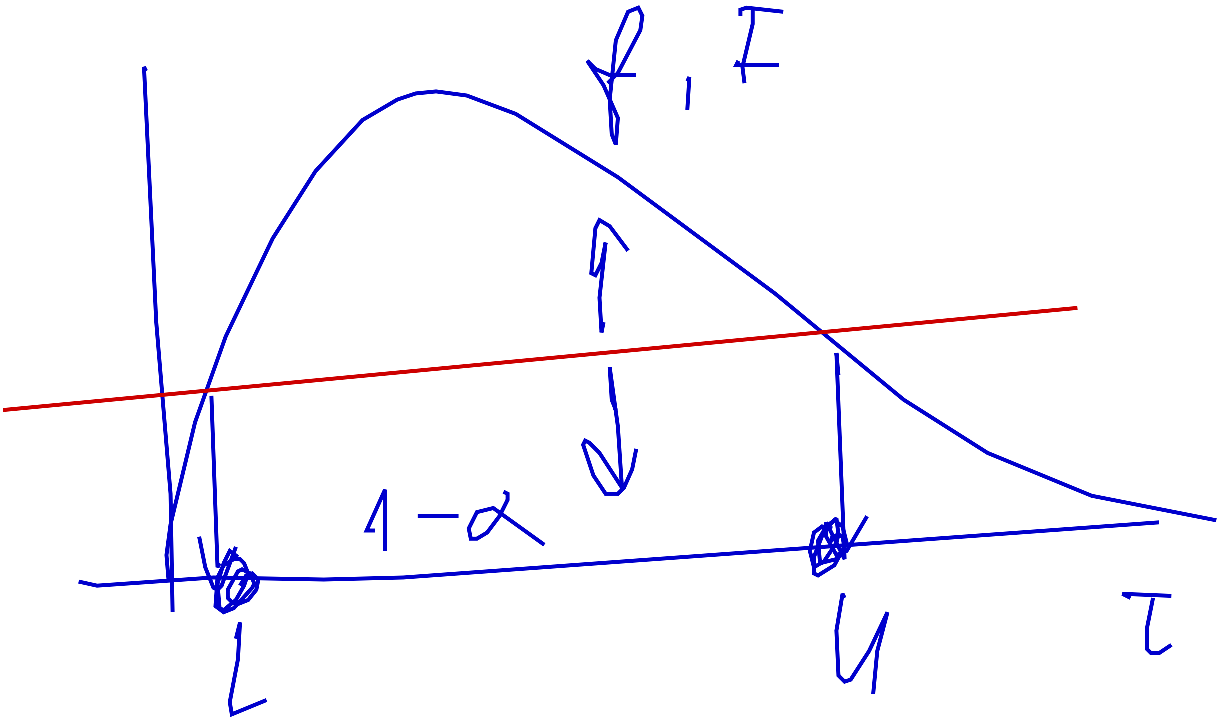
$$= \underline{p(\mu|\tau(\sigma),y)}\cdot \underline{p(\tau(\sigma)|y)|\tau'(\sigma)|}$$







HPD interval pro τ :



$$f(l) = f(u)$$

$$F(u) - F(l) = 1-\alpha$$

$$f(l) - f(u) = 0$$

$$F(u) - F(l) - 1 + \alpha = 0$$

$$\min_{(l,u)} |f(l) - f(u)|^2 + |F(u) - F(l) - 1 + \alpha|^2$$

$$p(\theta|y) = \frac{p(y|\theta)\,p(\theta)}{\int \ldots}$$

$$\theta^1, \ldots, \theta^M \sim p(\theta|y)$$

$$\int t(\theta)\, p(\theta|y)\, d\theta = E[t(\theta)]$$

$$\frac{1}{M}\sum_{m=1}^{M} t(\theta^m) \xrightarrow[M\to\infty]{s.i.} E[t(\theta)]$$

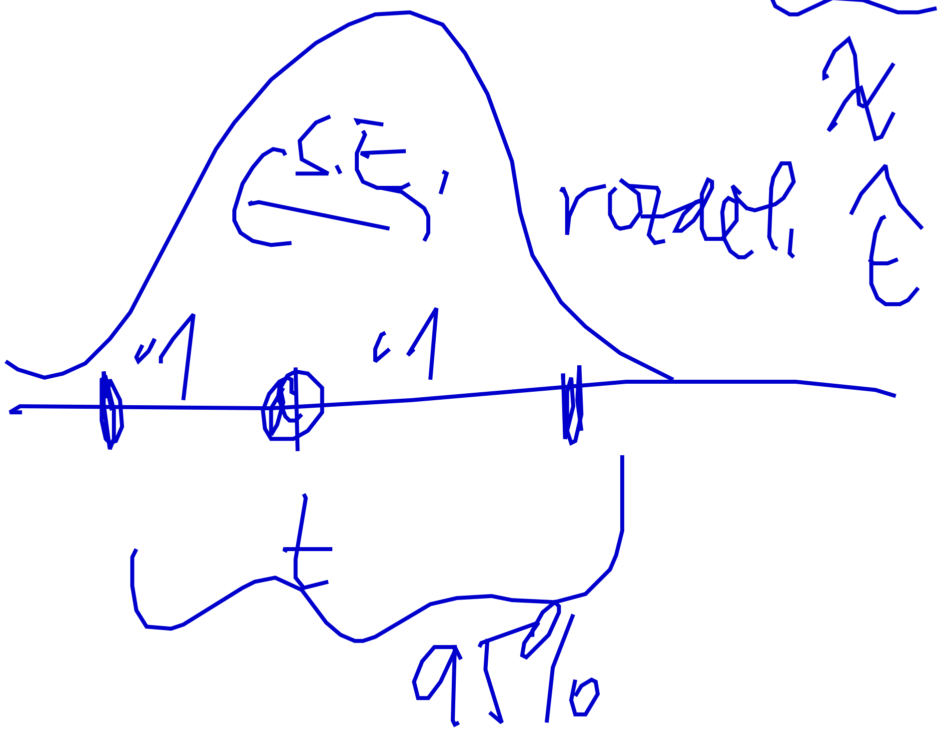
$$\operatorname{var}(t(\theta)) = \int t^2(\theta) \ldots - (E(\ ))^2$$

$$\underset{\substack{\downarrow\, n\to\infty \\ 0}}{S.E.(\hat{t})} \qquad \hat{t} \xrightarrow{P} t$$

+ asympt. normální

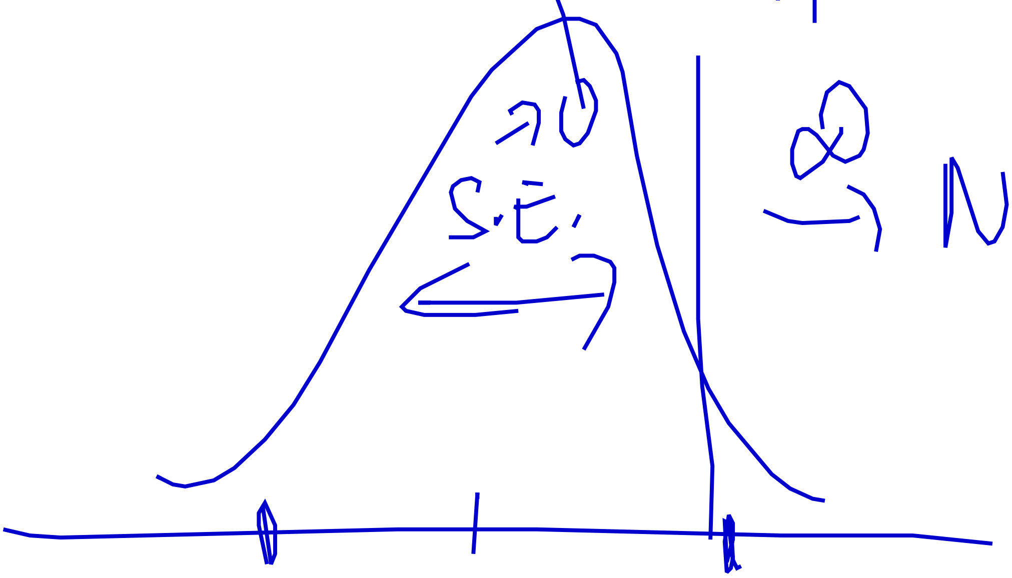
Interval spoleh.:

$$\hat{t} \pm \underbrace{SE(\hat{t})\, u_{1-\frac{\alpha}{2}}}_{\approx 2}$$



$\hat{t}_M$ rozdělení

$\overset{as}{\sim} N(Et, \omega)$



Et

$$\left(t \pm \hat{st} \cdot u_{1-\frac{\alpha}{2}}\right)$$

$p(\theta|y) \quad \theta \in \mathbb{R}^k, \; k \approx 10^4$

$$E(\theta_j|y) = \int \theta_j \, p(\theta|y) \, d\theta$$

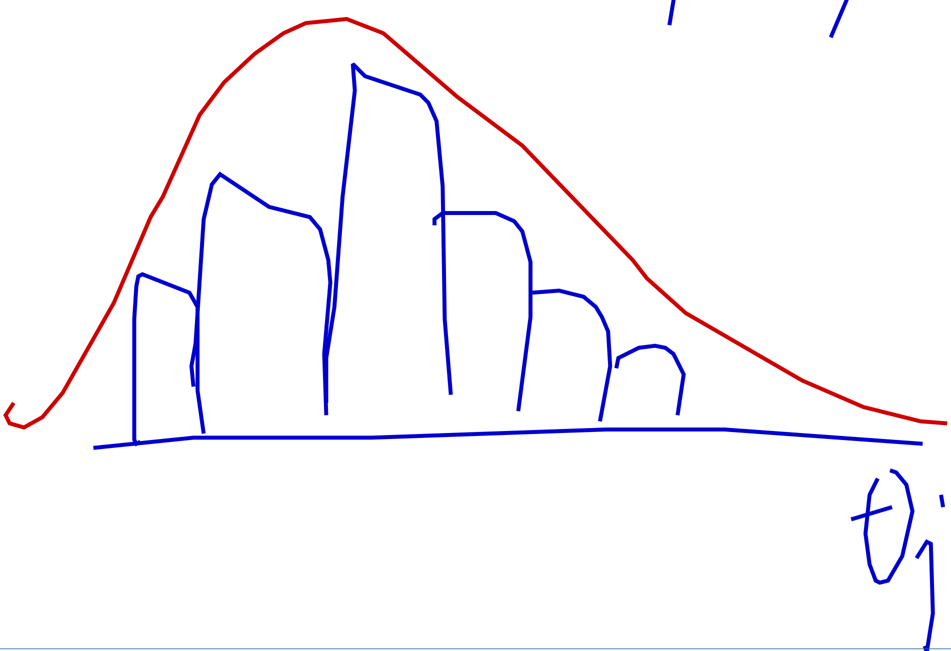
10^4 wym.

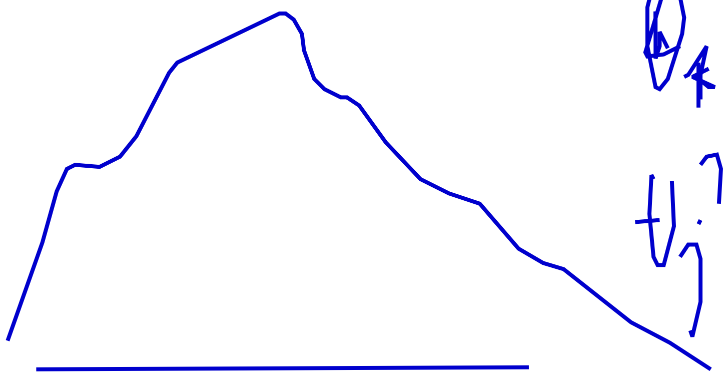
$$p(\theta_j|y) = \int p(\theta|y) \, d\theta_{(-j)}$$

$10^4 - 1$

$$F_{\theta_j}(x) = P(\theta_j \le x) =$$

$$= E\, I(\theta_j \le x)$$



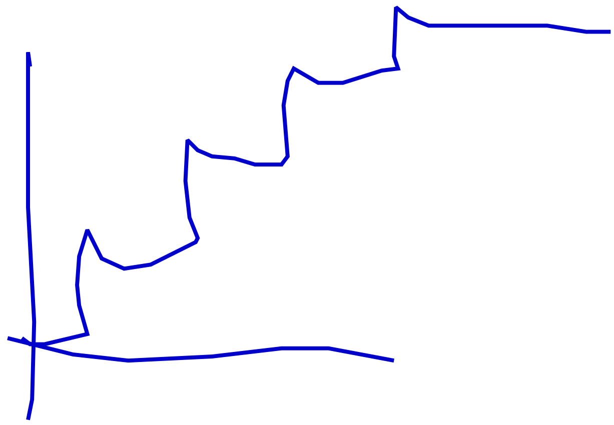


$\theta_j^1, \ldots, \theta^M$

$\theta_j^1, \ldots, \theta_j^M$

$$p(\theta_j|y) = \int p(\theta|y)\, d\theta_{-j}$$

θ



$r(\theta) :=$ odvozený parametr

$$P(r(\theta) \leq x \mid Y) =$$

$$= \frac{1}{n} \sum_{m=1}^{M} \mathbb{I}(r(\theta^m) \leq x)$$

$f_r(r) =$ transformace $p(\theta \mid y)$

$f :=$ aposteriorní hustota

$F :=$ ~~apo~~ aposteriorní distr. fce

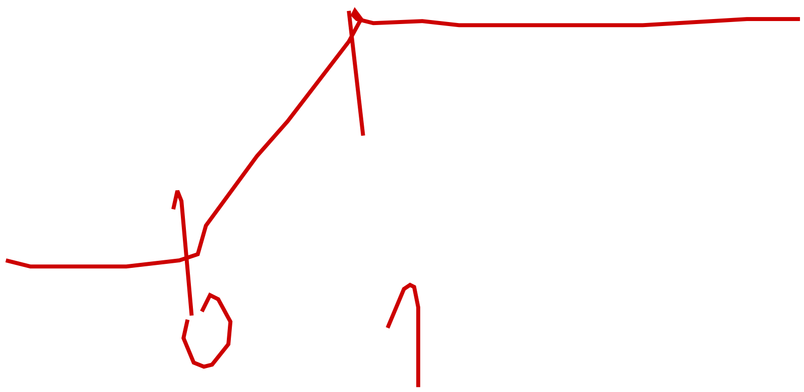
$U \sim U(0,1)$

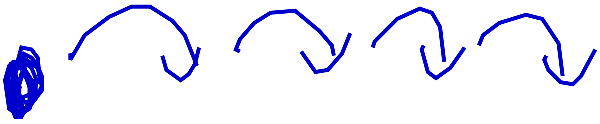
$\theta \sim F$, $F^{-1}(u) = \inf\{\theta : F(\theta) \geq u\}$

$Z = F^{-1}(U) \sim \theta$ (distr. der F)

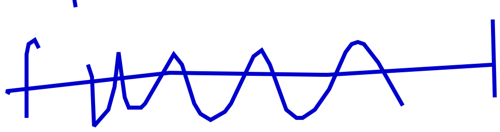
$P(Z \leq z) = P(F^{-1}(U) \leq z) =$

$= P(U \leq F(z)) = F(z)$





seed



$$F(x) = \int_{-\infty}^{x} f(s)\,ds$$

$$f(\theta_1, \dots, \theta_k) = f(\theta_1 \mid \theta_2, \dots, \theta_k)$$

$$\cdot f(\theta_2 \mid \theta_3, \dots, \theta_k) \cdot \dots$$

$$\dots f(\theta_{k-1} \mid \theta_k)\, f(\theta_k)$$

LM:

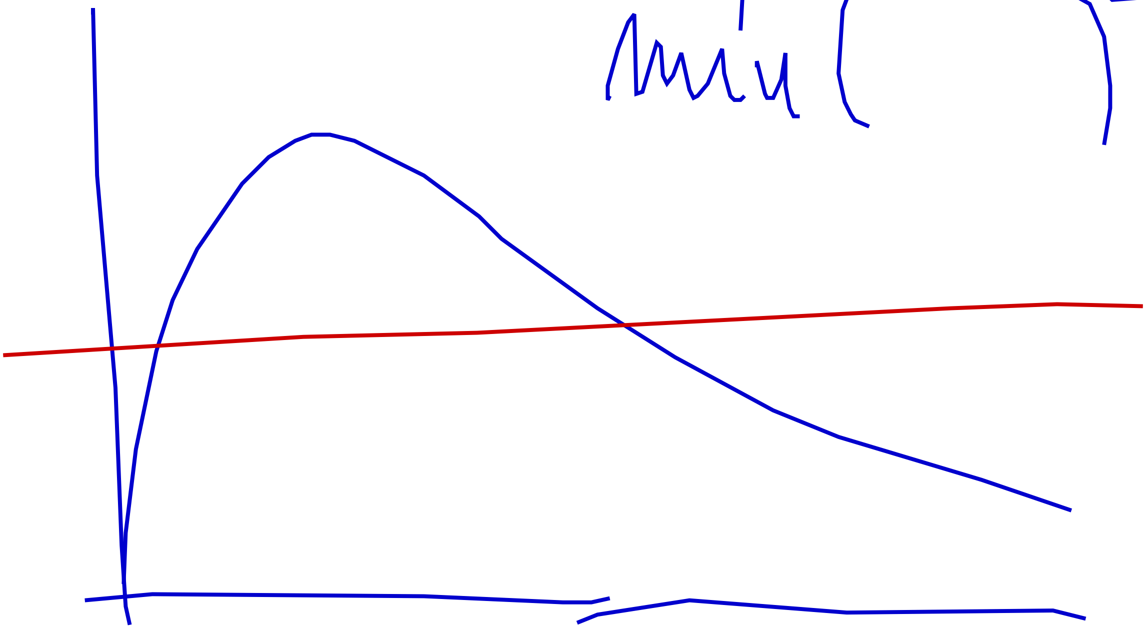
$$p(\beta, \tau \mid y) = p(\beta \mid \tau, y)\; p(\tau \mid y)$$

$$\sim N_k(\dots) \qquad \sim Gam(\dots)$$

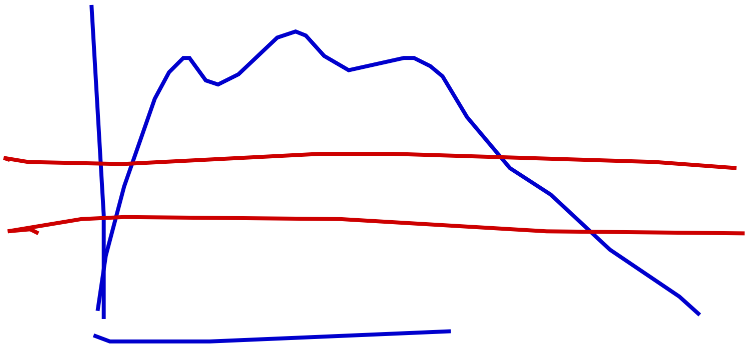
$$(\beta^1, \tau^1), (\beta^2, \tau^2), (\beta^3, \tau^3), \dots$$

$$| \dots \dots \backslash$$

$\min(\quad)^2 + (\quad)^T$



$\theta^1, \ldots, \theta^M$



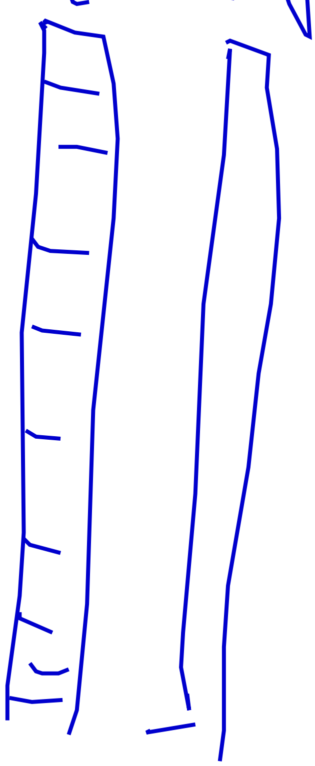
HPD interval (výběr)

MCE: $\hat{t} \pm MCE \cdot u_{1-\frac{\alpha}{2}}$

CVIČENÍ

```
for (m in 1:M){
    τ(m) <- rgamma(   )
    μ(m) <- rnorm( , τ(m))
}

τ <- rgamma(M, ⊙ ⊙)
        μ <- rnorm(M, ⊙, ⊙)
```



μ

$$p(y_{n+1}|y) = \int p(y_{n+1}, \theta | y)\, d\theta =$$

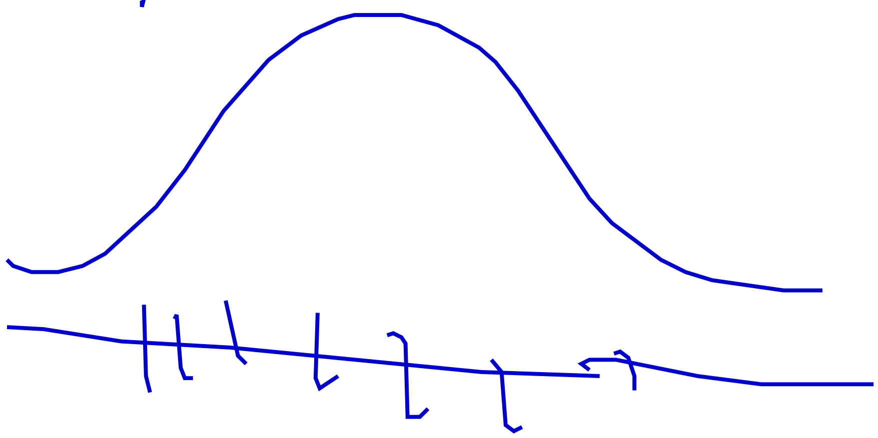
$$= \int p(y_{n+1}|\theta, \cancel{y})\, \underbrace{p(\theta|y)}_{\text{aposter.}}\, d\theta =$$

$Y_1, \dots, Y_n, Y_{n+1} | \theta$ v nezávislé

$$L(\theta) = \cancel{p(\theta|y)} \quad p(y|\theta) = \prod_{i=1}^{n} p(y_i|\theta)$$

$$= \int p(y_{n+1}|\theta)\, p(\theta|y)\, d\theta =$$

$$= E_{p(\theta|y)}\, p(y_{n+1}|\theta)$$

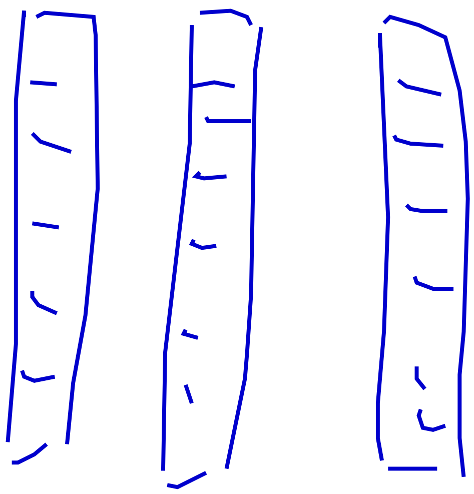


$p(y_{n+1}|y) \equiv$ marginální z

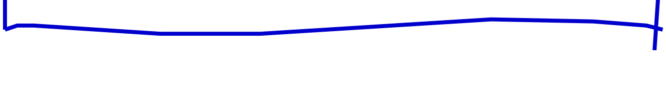
$p(y_{n+1},\theta|y) =$

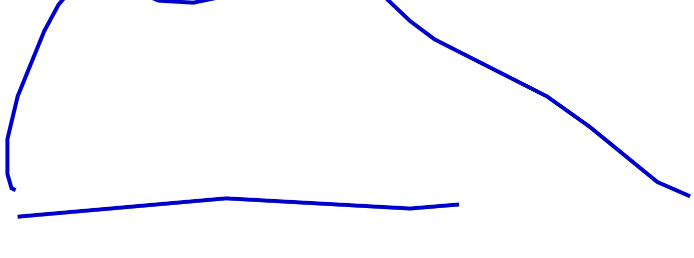
$= p(y_{n+1}|\theta,\cancel{y})\, p(\theta|y)$

$\tau \qquad \mu \qquad y_{n+1}$



$\tau|y \qquad \mu|\tau,y$





$$z \sim N(\mu, \Sigma)$$

$$\Sigma = LL^T$$

$$U \sim N(0, I_p)$$

$$z = \mu + LU$$

$U_1, \ldots, U_p \overset{iid}{\sim} N(0,1)$