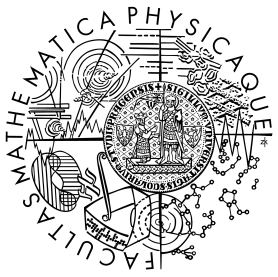



FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

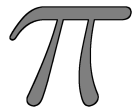
# Book of Abstracts

of the

**9th Day of Doctoral Students**

**of the School of Mathematics**

**June 13, 2023**



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# Preface

In the beginning of 2014, the Management of the Faculty of Mathematics and Physics decided that the traditional conference of PhD students called the WDS (Week of Doctoral Students) would not be organized as an activity of the entire faculty. Instead, the decision as to whether to organize the conference or not was left to the respective Schools (of Computer Science, of Mathematics, and of Physics).

Since then, the School of Mathematics organized this event as WDS-M (Week of Doctoral Students of the School of Mathematics). Except for 2014, WDS-M was always a one-day conference and therefore, the new name DDS-M (Day of Doctoral Students of the School of Mathematics) was introduced two years ago. Since WDS-M was not organized in 2020 due to the COVID-19 Pandemic, the conference of PhD students at the School of Mathematics is organized for the 9th time in this year, see also `http://www.karlin.mff.cuni.cz/~knobloch/DDS-M/2023/`. The original WDS continued at the School of Physics in its 32nd edition (May 30 – June 1, 2023) as a conference for PhD students of physical study programs, see `http://www.mff.cuni.cz/veda/konference/wds/`.

This year, 16 students have registered as active participants to the conference. They will present 13 lectures and 3 posters. The abstracts of the lectures are contained in this Book of Abstracts. We believe that this event, which takes place in the "mathematical" Karlín building of the faculty, will attract the attention of the students but also of the broad mathematical audience. We thus encourage all of those interested in the scientific activities of our doctoral students to attend this meeting.

The conference is co-organized by the *School of Mathematics, Faculty of Mathematics and Physics, Charles University*, and *Charles University Chapter of SIAM*.

Prague, June 13, 2023

doc. Mgr. Petr Knobloch, Dr., DSc.
Coordinator of the Doctoral Studies at the School of Mathematics
Faculty of Mathematics and Physics
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# Limits of structures and total NP search problems

**Mgr. Ondřej Ježil**

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*Year of study:* 1
*Supervisor:* prof. RNDr. Jan Krajíček, DrSc.

**Abstract**

For a class of finite graphs, we define a limit object relative to some computationally restricted class of functions. The properties of the limit object then reflect how a computationally restricted viewer "sees" a generic instance from the class. The construction uses Krajíček's forcing with random variables. We provide means to calculate truth-values of simply stated properties, and prove that such a limit object can then be expanded to a model of weak arithmetic. We then take a limit of all finite pointed paths and expand it to a model of arithmetic, such that the properties of resulting structure reflect the properties of randomized oracle computations.

# Logic, complexity, search problems and games

**Mgr. Mykyta Narusevych**

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*Supervisor:* prof. RNDr. Jan Krajíček, DrSc.

**Abstract**

The goal of my talk is to show connections between logic and complexity theory via the bounded arithmetic. I will discuss the notion of a black-box PLS model and show its connection to the theory $T_2^1$. In particular, I will show a computational problem which is not black-box PLS solvable. Finally, I will present our ongoing research project connected to certain combinatorial game and mention several open problems.

# Ideal lattices in cryptography

**Mgr. Sára Šůstek Vyhnalová**

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*Supervisor:* doc. Mgr. Pavel Příhoda, Ph.D.

**Abstract**

In the lecture, we will introduce the concept of lattices as an algebraic structure along with the cryptographic problems based on lattices, namely SVP (Shortest Vector Problem) and CVP (Closest Vector Problem). We will discuss the threat to current Asymmetric Cryptography posed by quantum computers and one of the alternatives, which is lattice-based cryptography. We will also explore a more efficient structure known as ideal lattices. Finally, we will present current problems and the goals we aim to achieve in our research.

# Tverberg-type problems and some recent proceedings

**Mgr. Yifan Zhang**

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**Abstract**

A central unsolved problem in the area of topological combinatorics is the famous topological Tverberg conjecture, which claims that for any continuous map $f$ from a $(d+1)(r-1)$-dimensional simplex $\Delta$ to the $d$-dimensional Euclidean space $\mathbb{R}^d$, there exist pairwise disjoint faces $\sigma_1, \cdots, \sigma_r$ of $\Delta$ such that $f(\sigma_1) \cap \cdots \cap f(\sigma_r) \neq \varnothing$, where $r \geq 2$ and $d \geq 1$ are any integers. Originally proven for $r$ a prime power, recent developments show that counterexamples exist in other cases for sufficiently large $d$ with respect to $r$. In this talk, a concise introduction to the background of Tverberg-type problems is given, as well as a brief review of some recent progress about the topological Tverberg conjecture, so that one may appreciate the fruitful interaction between different branches of mathematics contained in the arguments.

# Topological fractals

**Mgr. Klára Karasová**

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**Abstract**

A continuum is a nonempty connected compact metric space. Peano continuum is a locally connected continuum. We say that a compact metrizable space $X$ is a topological fractal if there exist finitely many continuous maps $f_1, f_2, \ldots, f_n : X \to X$ whose images cover $X$ such that the system $\{f_1, \ldots, f_n\}$ is topologically contractive. In 1985, Hata observed that every connected topological fractal is a Peano continuum and asked whether every Peano continuum is a topological fractal. His question remains open, nevertheless recently was proved that every Peano continuum containing a free arc is a topological fractal and this result has been significantly strengthen already. Jointly with B. Vejnar we give another sufficient condition under which Peano continuum is a topological fractal, namely uncountably many local cut-points.

# The homological origin of Feynman diagrams

**Mgr. Martin Zika**

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*Year of study:* 1
*Supervisor:* prof. Ing. Branislav Jurčo, CSc., DSc.

**Abstract**

Feynman's diagrammatic techniques are ubiquitous in quantum field theory, have contributed to some of the most precise scientific predictions ever and found its way even to pop culture. They are however much more universal; we can forget about the particle interpretation and use them to organize perturbative expansions of integrals with a weight $\exp(S)$, where $S$ has a non-degenerate Hessian. We will discuss how this can be further abstracted to the language of homological algebra and hint at why this is the key to describe the homotopy theory of quantum $L_\infty$ algebras.

# Lipschitz free spaces and subsets of finite-dimensional spaces

**Mgr. Jan Bíma**

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*Year of study:* 1
*Supervisor:* doc. Mgr. Marek Cúth, Ph.D.

**Abstract**

Given a pointed metric space $\mathcal{M}$ and $0 < p \leq 1$, there exists a $p$-Banach space $\mathcal{F}_p(\mathcal{M})$, called the *Lipschitz free $p$-space over* $\mathcal{M}$, such that $\mathcal{M}$ embeds isometrically into $\mathcal{F}_p(\mathcal{M})$, and for every Banach $p$-space $Y$ and a Lipschitz map $f : \mathcal{M} \to Y$ which vanishes at the distinguished point of $\mathcal{M}$, $f$ extends naturally and uniquely to a linear operator $T_f : \mathcal{F}_p(\mathcal{M}) \to Y$, whose norm equals the Lipschitz constant of $f$. In my talk, I will motivate the study of Lipschitz free spaces and discuss their relevance as a bridge between the classical linear theory and the non-linear geometry of Banach spaces. The main focus of the talk will be on the unique challenges arising from the locally non-convex geometry of Lipschitz free $p$-spaces for $0 < p < 1$, as well as on some related open questions and recent advancements in the theory of Lipschitz free $p$-spaces over subsets of finite-dimensional spaces.

# Mazur-Ulam property of unital JB*-algebras

**Mgr. Radovan Švarc**

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*Study branch:* P4M3
*Year of study:* 3
*Supervisor:* prof. RNDr. Ondřej Kalenda, Ph.D., DSc.

**Abstract**

JB*-algebras are an interesting mathematical object, fusing the metric structure of Banach spaces with algebraic structure of Jordan's algebras. Tingley's problem is an important open question in the field of Banach spaces, asking the following question: consider any two Banach spaces $X$ and $Y$ such that their unit spheres $S_X$ and $S_Y$ are isometric via mapping $T : S_X \to S_Y$. Can this $T$ be always extended to a real-linear isometry mapping $X$ to $Y$? If for a given $X$ the answer is positive for any Banach space $Y$, we say that $X$ has the Mazur-Ulam property. In the talk we will discuss the proof of this property for any unital JB*-algebra.

# Error analysis of a time-continuous numerical scheme for degenerate parabolic problems

**M.Sc. Sunčica Sakić**

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*Year of study:* 3
*Supervisor:* Scott Congreve, Ph.D.

**Abstract**

In this talk we present an error analysis for the spatial discretization of Richards' equation. The Richards' equation is a doubly nonlinear parabolic partial differential equation which can degenerate to an elliptic equation or ordinary differential equation in real applications such as porous media flows. Therefore, it is challenging to develop and analyze a sufficiently accurate and efficient method for its numerical solution.

We transform the original problem using (expanded) mixed formulation and define the local discontinuous Galerkin method to discretize the spatial variable while the time remains continuous. We derive error estimates in terms on Holder coefficient of nonlinear temporal derivative function and spatial discretization parameter. Moreover, the theoretical results are supported by numerical experiments.

# Bootstrapping not independent and not identically distributed data

**Mgr. Martin Hrba**

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*Year of study:* 2
*Supervisor:* doc. RNDr. Michal Pešta, Ph.D.

**Abstract**

Classical normal asymptotics could bring serious pitfalls in statistical inference, because some parameters appearing in the limit distributions are unknown and, moreover, complicated to estimate (from a theoretical as well as computational point of view). Due to this, plenty of stochastic approaches for constructing confidence intervals and testing hypotheses cannot be directly applied. Bootstrap seems to be a plausible alternative. A methodological framework for bootstrapping not independent and not identically distributed data will be presented together with theoretical justification of the proposed procedures and an example of application to data in insurance.

# Non-smooth stochastic systems

**Mgr. František Hendrych**

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*Year of study:* 1
*Supervisor:* RNDr. Petr Čoupek, Ph.D.

**Abstract**

Solving stochastic differential equations pathwise is problematic in general. The aim of this talk is to present general concept of Rough Path Theory which can be useful in many such situations.

# CLT for Poisson-Laguerre tessellation with unbounded weights

**Mgr. Martina Petráková**

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*Year of study:* 1
*Supervisor:* doc. RNDr. Zbyněk Pawlas, Ph.D.

**Abstract**

Laguerre tessellation is a generalisation of the well-known Voronoi tessellation that uses power distance, instead of the standard Euclidean, to determine the cell that belongs to the generating weighted point. To randomise this object, we can consider the set of generating points to be random. Particularly, we consider the Poisson-Laguerre tessellation, where the random generator is a Poisson marked point process with marks representing the weights.

As the exact distributions of the functionals of the Poisson-Laguerre tessellation are hard to derive, except for trivial cases, the primary concern is to examine their asymptotic properties. Several central limit theorems were derived using the stabilisation method, which forced the assumption of bounded weights. The goal of the current research is to use the Malliavin–Stein method and the concept of tempered configurations to derive central limit theorems for functionals of the Poisson-Laguerre tessellation with unbounded weights.

# Incorporating inertial dynamics in the Vicsek model

**Mgr. David Voráč**

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*Year of study:* 1
*Supervisor:* RNDr. Artem Ryabov, Ph.D.

**Abstract**

Efficient global response in many-body systems like bird flocks and crowds requires rapid information propagation. Our study reveals the relationship between local perturbations and collective behavior in highly polarized networks described by various variants of the Vicsek model. We find that information spreads faster in the model version with orientational inertia. However, we also showed that the overdamped and inertial variants of the model share many similarities.

# Posters

## Shelah's categoricity conjecture for AEC's of modules

**Mgr. Kateřina Fuková**

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## Future considerations based on the weighted Nadrajah-Haghighi distribution

**Mgr. Muhammad Nauman Khan**

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## New approaches based on matrix equations for the solution of non-autonomous linear ODEs

**Mgr. Faisal Mahmood**

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