Name:

Convex optimization

Homework set 2

Date due: October 25 2019, 10:41

Explain your reasoning in all the problems.

Problem	Pts max	Pts
1	2	
2	2	
3	2	
4	2	
5	2	
Σ	10	

Problem 1. Sketch two points $\mathbf{x}, \mathbf{y}$ in $\mathbb{R}^2$ that are incomparable (in the generalized inequality with respect to $\mathbb{R}^2_+$), i.e. neither $\mathbf{x} \preceq \mathbf{y}$, nor $\mathbf{y} \preceq \mathbf{x}$ holds. Briefly explain your reasoning.

Problem 2. Let K be a proper cone in $\mathbb{R}^n$ and $\mathbf{x} \in \mathbb{R}^n$. Prove $\mathbf{x}, -\mathbf{x} \succeq_K \mathbf{0}$ if and only if $\mathbf{x} = \mathbf{0}$.

Problem 3. Prove in detail that if $f\colon \mathbb{R}^k \to \mathbb{R}^n$ is an affine function and $X \subset \mathbb{R}^n$ is convex then $f^{-1}(X)$ is a convex set.

Hint: Affine functions "commute" with convex combinations.

Problem 4. Prove that every $A \in S_+^n$ is a conical combination of matrices of the form $\mathbf{v}\mathbf{v}^T$. That is, show that for every $A \in S_+^n$ there exist vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n \in \mathbb{R}^n$ and nonnegative numbers $\lambda_1, \ldots, \lambda_n$ such that

$$A = \lambda_1 \mathbf{v}_1 \mathbf{v}_1^T + \lambda_2 \mathbf{v}_2 \mathbf{v}_2^T + \cdots + \lambda_n \mathbf{v}_n \mathbf{v}_n^T.$$

Hint: Look at eigenvalues of positive semidefinite matrices.

Problem 5. Write a program in Python using the CVXOPT/CVXPY library that solves the following problem (from the first tutorial; values slightly changed):

We are ordering natural gas from the gas company for the winter of 2019. The way it works, we pre-order x cubic meters of gas for 8 Kč/m^3 and for every cubic meter above this amount we pay 15 Kč. We pay for the x cubic meters in advance and any unused gas is lost (not refunded).

We have a model that says that this year's winter will be the same as one of the previous ten winters (each has probability 10% to repeat); the historic heat consumption is as follows:

Winter	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009
m^3	80	50	200	70	60	20	300	150	180	100

Model the problem as a linear programming problem that you will then solve in Python using one of the libraries. Explain what your general idea is and note the optimal x you got here on paper and *send* your code by the due date to Jiří to `pavluji@artax.karlin.mff.cuni.cz`

You can consult with your friends when solving the homework, but you have to **write** your solutions (including Python code) **on your own** and **do not show your fininished solutions** to your peers before the due date.