Convex optimization

Tutorial 6

Problem 1. Rewrite the LP

$$\begin{aligned} &\text{minimize } x_1 - x_2 + 2x_3 \\ &\text{subject to } x_1 + x_2 + x_3 \leq 10 \\ &x_1 \geq 0 \\ &x_2 - x_3 = 1. \end{aligned}$$

To an equivalent problem in the standard form:

$$\begin{aligned} &\text{minimize } \mathbf{c}^T \mathbf{y} \\ &\text{subject to } \mathbf{y} \succeq 0, \\ &A\mathbf{y} = \mathbf{b}. \end{aligned}$$

Problem 2. Let P be an LP in the standard form (see above). How would you construct an SDP (in any of the three forms we had) that is equivalent to P?

Problem 3 (Robust LP)**.** Suppose that in the ice cream problem from before we do not quite know what the demand will be each month. Instead, we have a vector $\mathbf{b} \in \mathbb{R}^{12}$ and a matrix $A \in S^{12}_{++}$ such that with 95 % probability the ice cream demand vector in different months, $\mathbf{z} \in \mathbb{R}^{12}$, satisfies $(\mathbf{z}-\mathbf{b})^T A(\mathbf{z}-\mathbf{b}) \leq 1$.

Formulate an SOCP for the problem "plan production amounts $p_1, \ldots, p_{12}$ to satisfy all demand with probability 95 % with minimal cost" (recall that we pay $c > 0$ for storing 1 unit of ice cream 1 month and $s > 0$ for a month-to-month change of production).

Problem 4. Let X, Y be symmetric $n \times n$ matrices. Prove that $X \succeq Y$ if and only if for all vectors $\mathbf{x} \in \mathbb{R}^n$ we have $\mathbf{x}^T X \mathbf{x} \geq \mathbf{x}^T Y \mathbf{x}$.

Problem 5. Let $n \in \mathbb{N}$. Prove that the interior of S^n_+ (in the space of symmetric matrices) is exactly S^n_{++}.