Written Exam on Mathematics II for IES FSV UK (E)

Summer Semester 2012-2013

Problem 1: Compute the inverses of the matrices $\mathbb{A}$ and $\mathbb{B}$, where $\mathbb{A}$ is given below and $\mathbb{B}$ has in the first row $\frac{1}{4}$ of the fourth row of $\mathbb{A}$, in the second row $\frac{1}{4}$ of the third row of $\mathbb{A}$, in the third row the second row of $\mathbb{A}$ and in the fourth row the first row of $\mathbb{A}$.

$$\mathbb{A} = \begin{pmatrix} 5 & 0 & 0 & 1 \\ 0 & 1 & 5 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 5 & 5 & 1 \end{pmatrix} \qquad \text{(10 points)}$$

Problem 2: Determine and draw the domain of the function

$$f(x,y) = \arcsin(x^2 + y^2 - 2),$$

compute its partial derivatives with respect to all the variables at all points where they exist.

(10 points)

Problem 3: Let us consider the equation $4\,\mathrm{arctg}(xy) + \sin(x+y) + \pi = 0$ and the point $[-1,1]$. Show that this equation defines a C^∞ function $y = f(x)$ defined on a neighborhood of -1, which satisfies $f(-1) = 1$. Compute $f'(-1)$, $f''(-1)$ and determine the equation of the tangent line to the graph of f at the point $[-1, f(-1)]$. (10 points)

Problem 4: Determine sup and inf of the function f on the set M and decide whether these values are attained, if

$$f(x,y,z) = x - yz \text{ and } M = \left\{[x,y,z] \in \mathbb{R}^3 : x^2 + 3y^2 + z^2 = 4, z + y\sqrt{3} + 1 \leq 0\right\} \qquad \text{(15 points)}$$

Problem 5: Compute the following antiderivative on maximal possible intervals:

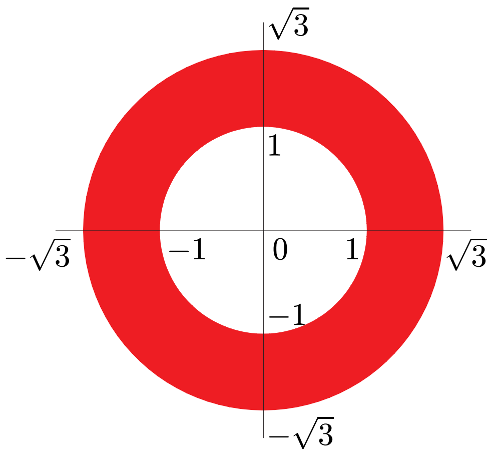
$$\int \frac{x^4 + 24}{(x+2)(x^3 - 8)}\,\mathrm{d}x \qquad \text{(15 points)}$$

Answers to the Written Exam on Mathematics II for IES FSV UK (E)
Summer Semester 2012-2013

Problem 1: $\mathbb{A}^{-1} = \begin{pmatrix} \frac{1}{4} & 0 & -\frac{5}{16} & \frac{1}{16} \\ 0 & -\frac{1}{4} & -\frac{5}{16} & \frac{5}{16} \\ 0 & \frac{1}{4} & \frac{1}{16} & -\frac{1}{16} \\ -\frac{1}{4} & 0 & \frac{25}{16} & -\frac{5}{16} \end{pmatrix}$, $\mathbb{B}^{-1} = \begin{pmatrix} \frac{1}{4} & -\frac{5}{4} & 0 & \frac{1}{4} \\ \frac{5}{4} & -\frac{5}{4} & \frac{1}{4} & 0 \\ \frac{-}{1}4 & \frac{1}{4} & \frac{1}{4} & 0 \\ -\frac{5}{4} & \frac{25}{4} & 0 & -\frac{1}{4} \end{pmatrix}$.

Problem 2: $D_f = \{[x,y] \in \mathbf{R}^2 : 1 \le x^2 + y^2 \le 3\}$.

Picture of the domain:



$\frac{\partial f}{\partial x}(x,y) = \frac{2x}{\sqrt{1-(x^2+y^2-2)^2}}$ and $\frac{\partial f}{\partial y}(x,y) = \frac{2x}{\sqrt{1-(x^2+y^2-2)^2}}$; both partial derivatives on the set $\{[x,y] \in \mathbf{R}^2 : 1 < x^2 + y^2 < 3\}$.
At points $[x,y]$ satisfying $x^2 + y^2 = 3$ the partial derivatives have no sense, since there is neither horizontal nor vertical segment centered at that point and contained in D_f. At points $[x,y]$ satisfying $x^2 + y^2 = 1$ it make sense to compute $\frac{\partial f}{\partial x}(0,1)$, $\frac{\partial f}{\partial x}(0,-1)$, $\frac{\partial f}{\partial y}(1,0)$ and $\frac{\partial f}{\partial y}(-1,0)$, for the remaining points there is no horizontal (resp. vertical) segment centered at that point and contained in D_f. The mentioned four partial derivatives do not exist.

Problem 3: $f'(-1) = 3$, $f''(-1) = 20$, tangent line $y = 1 + 3(x+1)$.

Problem 4: Maximum $\sqrt{3}$ at the points $[\sqrt{3}, 0, -1]$ and $[\sqrt{3}, -\frac{1}{\sqrt{3}}, 0]$, minimum $-\frac{7}{6}\sqrt{3}$ at the point $[-\sqrt{3}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{2}}]$.

Problem 5: $\int \frac{x^4+24}{(x+2)(x^3-8)}\,\mathrm{d}x \stackrel{c}{=} x - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| - \frac{1}{6}\log(x^2+2x+4) - \sqrt{3}\,\mathrm{arctg}\,\frac{x+1}{\sqrt{3}}$ on each of the three intervals $(-\infty,-2)$, $(-2,2)$ and $(2,+\infty)$.