Written Exam on Mathematics II for IES FSV UK (D)

Summer Semester 2012-2013

Problem 1: Find all the solutions of the system $\mathbb{A}\mathbf{x} = \mathbf{b}$ for the below given matrix $\mathbb{A}$ and given three right-hand side vectors $\mathbf{b}_1$, $\mathbf{b}_2$ a $\mathbf{b}_3$.

$$\mathbb{A} = \begin{pmatrix} 5 & 0 & 2 & -7 \\ -7 & 0 & 2 & 5 \\ 5 & 2 & 0 & -7 \\ 0 & 2 & 5 & -7 \end{pmatrix}, \mathbf{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{b}_2 = \begin{pmatrix} 7 \\ -5 \\ 2 \\ 2 \end{pmatrix}, \mathbf{b}_3 = \begin{pmatrix} -2 \\ -2 \\ 10 \\ 5 \end{pmatrix}$$

(10 points)

Problem 2: Determine and draw the domain of the function

$$f(x,y) = (x^2 + y + 1)^{|x+y|},$$

compute its partial derivatives with respect to all the variables at all points where they exist.

(10 points)

Problem 3: Let us consider the equation

$$\log(x + 2y^2) + e^{x+y} - 1 = 0$$

and the point $[-1, 1]$. Show that this equation defines a C^∞ function $y = f(x)$ defined on a neighborhood of -1, which satisfies $f(-1) = 1$. Compute $f'(-1)$, $f''(-1)$ and determine the equation of the tangent line to the graph of f at the point $[-1, f(-1)]$. (10 points)

Problem 4: Determine sup and inf of the function f on the set M and decide whether these values are attained, if

$$f(x,y,z) = x + y + z \text{ and } M = \{[x,y,z] \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 12, xy \geq 5\}$$

(15 points)

Problem 5: Compute the following antiderivative on maximal possible intervals:

$$\int \frac{4x^4}{(x+1)^2(x^2+2x+3)} \, \mathrm{d}x$$

(15 points)

Answers to the Written Exam on Mathematics II for IES FSV UK (D)
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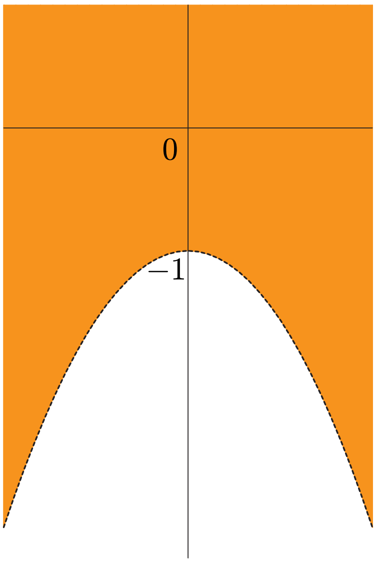
Problem 1: For $\mathbf{b}_1$: no solution. For $\mathbf{b}_2$: infinitely many solutions of the form $[t+1, t-\frac{3}{2}, t+1, t]$, $t \in \mathbf{R}$. For $\mathbf{b}_3$: infinitely many solutions of the form $[t, t+5, t-1, t]$, $t \in \mathbf{R}$.

Problem 2: $D_f = \{[x,y] \in \mathbf{R}^2 : y > -x^2 - 1\}$.

Picture of the domain:



$\frac{\partial f}{\partial x}(x,y) = (x^2+y+1)^{|x+y|} \cdot (\operatorname{sgn}(x+y) \cdot \log(x^2+y+1) + |x+y| \cdot \frac{2x}{x^2+y+1})$ and $\frac{\partial f}{\partial y}(x,y) = (x^2+y+1)^{|x+y|} \cdot (\operatorname{sgn}(x+y) \cdot \log(x^2+y+1) + |x+y| \cdot \frac{1}{x^2+y+1})$; both partial derivatives on the set $\{[x,y] \in \mathbf{R}^2 : y > -x^2 - 1 \;\&\; x+y \neq 0\}$ All the points $[x,-x]$, $x \in \mathbf{R}$, belong to D_f and D_f is an open set, so it makes sense to compute partial derivatives at these points. At the points $[0,0]$ and $[1,-1]$ both partial derivatives are equal to zero, at the other points (i.e., at points $[x,-x]$, $x \in \mathbf{R} \setminus \{0,1\}$) the partial derivatives do not exist.

Problem 3: $f'(-1) = -\frac{2}{5}$, $f''(-1) = -\frac{16}{125}$, tangent line $y = 1 - \frac{2}{5}(x+1)$.

Problem 4: Maximum $\sqrt{2} + 2\sqrt{5}$ at the point $[\sqrt{5}, \sqrt{5}, \sqrt{2}]$, minimum $-\sqrt{2} - 2\sqrt{5}$ at the point $[-\sqrt{5}, -\sqrt{5}, -\sqrt{2}]$.

Problem 5: $\int \frac{4x^4}{(x+1)^2(x^2+2x+3)}\,\mathrm{d}x \stackrel{c}{=} 4x - \frac{2}{x+1} - 8\log|x+1| - 4\log(x^2+2x+3) + 7\sqrt{2}\operatorname{arctg}\frac{x+1}{\sqrt{2}}$ on each of the two intervals $(-\infty,-1)$, and $(-1,+\infty)$.