Malthusiánský model:

$y' = ay \qquad (a > 0) \qquad h(x) = 1$

$g(y) = ay$

① řešíme na $\mathbb{R}$

② $y = 0, \; x \in \mathbb{R}$ je stac. řešení

③ $(-\infty, 0), \; (0, +\infty)$

④ řešení nenabývající 0:

$$\frac{y'}{y} = a$$

$$\log|y| = ax + C$$

⑤ $G(y) = \log|y| \qquad G((-\infty,0)) = \mathbb{R}$

$G((0,+\infty)) = \mathbb{R}$

$\lim_{y \to 0-} \log|y| = -\infty$

$\lim_{y \to +\infty} \log|y| = +\infty$

$$|y(x)| = e^{ax+C}$$

$$|y(x)| = \underbrace{e^{C}}_{\in (0,+\infty)} \cdot e^{ax}$$

$$|y(x)| = k \cdot e^{ax} \quad (k > 0)$$

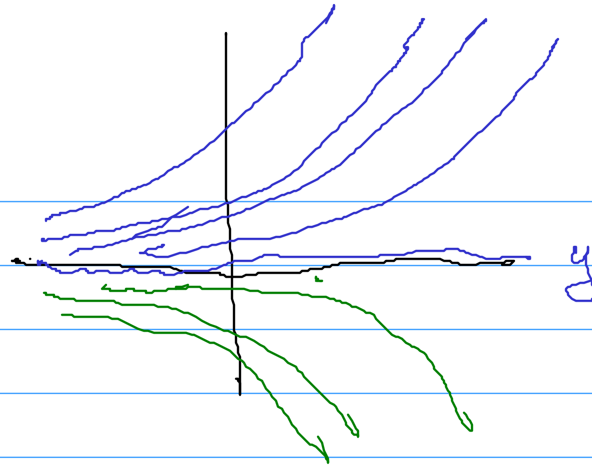
$$y(x) = \pm k e^{ax}$$

$$y(x) = k \cdot e^{ax} \quad (k \in \mathbb{R} \setminus \{0\}) \quad x \in \mathbb{R}$$

neprodloužitelné

Závěr: Max. řešení jsou: $y(x) = k \cdot e^{ax}$, $x \in \mathbb{R}$ $(k \in \mathbb{R})$

$k = y(0)$



$a > 0$ .... ↗

$a < 0$ .... y padá stejný

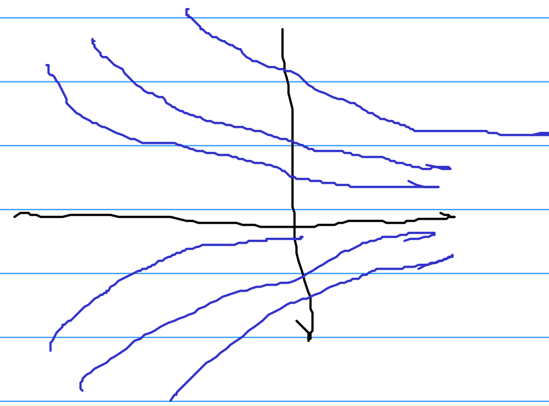
výsledek stejný $y(x) = k \cdot e^{ax}$

a = porodnost − úmrtnost

$a = 0$

$y' = 0$

$y = c$



Logistický model:

$$y' = ay - by^2 = y(a - by) \qquad (a, b > 0)$$

① max

② stac. řešení: $y = 0$, $y = \frac{a}{b}$ (max)

③ $(-\infty, 0)$, $(0, \frac{a}{b})$, $(\frac{a}{b}, +\infty)$

④ $$\frac{y'}{y(a-by)} = 1$$

$$\frac{1}{y(a-by)} = \frac{1}{a}\left(\frac{1}{y} + \frac{1\cdot b}{a-by}\right)$$

$$\frac{1}{a}\left(\lg|y| - \lg|a-by|\right) = x + C$$

$$\frac{1}{a}\lg\left|\frac{y}{a-by}\right| = x + C$$

(5)

$(0, \frac{a}{b}) \to \mathbb{R}$ (în $0+$ limita $-\infty$, în $\frac{a}{b}-$ limita $+\infty$)

$(\frac{a}{b}, +\infty) \to \left(\frac{1}{a}\lg\frac{1}{b}, +\infty\right)$ (în $\frac{a}{b}+$ limita $+\infty$, în $+\infty$ limita $\frac{1}{a}\lg\frac{1}{b}$)

$(0, \frac{a}{b})$:

$$\left|\overbrace{\frac{y}{a-by}}^{>0}\right| = e^{a(x+C)}$$

$$\frac{y}{a-by} = e^{a(x+C)}$$

$$y = (a-by)e^{a(x+C)}$$

$$y\left(1+be^{a(x+C)}\right) = ae^{a(x+C)}$$

$$y(x) = \frac{ae^{a(x+C)}}{1+be^{a(x+C)}}, \quad x \in \mathbb{R}$$

$y \in (\frac{a}{b}, +\infty)$

$$\frac{y}{a-by} = -e^{a(x+C)} \qquad x \in \left(\frac{1}{a}\lg\frac{1}{b} - C, +\infty\right)$$

$$y(x) = \frac{-ae^{a(x+C)}}{1-be^{a(x+C)}},$$