II.2 Multifunctions admitting unrestricted continuation

Definition. Let $\boldsymbol{f}$ be an analytic multifunction in a domain Ω.

- Let $G \subset \Omega$ be a domain. We say that $\boldsymbol{f}$ **admits unrestricted continuation in** G if whenever $(f, D) \in \boldsymbol{f}$ satisfies $D \subset G$ and $\gamma : [0,1] \to G$ is a continuous curve such that $\gamma(0)$ is the center of D, then (f, D) admits an analytic continuation along γ in G.
- If $\boldsymbol{f}$ admits unrestricted continuation in Ω, we say just that $\boldsymbol{f}$ **admits unrestricted continuation**.

Theorem 3. *Let $\boldsymbol{f}$ be an analytic multifunction in a domain Ω, which admits unrestricted continuation. Then there exists $p \in \mathbb{N} \cup \{\infty\}$, such that f is precisely p-valued.*

Theorem 4. *Nechť $\boldsymbol{f}$ be a singlevalued analytic multifunction in a domain Ω. Then*

- *there is $f \in H(\operatorname{dom}(\boldsymbol{f}))$ such that*

$$\boldsymbol{f} = \{(f, D) : D \text{ is a disc such that } D \subset \operatorname{dom}(\boldsymbol{f})\},$$

  and
- *$\boldsymbol{f}$ admits unrestricted continuation in $\operatorname{dom}(\boldsymbol{f})$.*

Lemma 5. *Let $\boldsymbol{f}$ be an analytic multifunction in a domain Ω, which admits unrestricted continuation in a domain $G \subset \Omega$. Let $(f, D) \in \boldsymbol{f}$ satisfy $D \subset G$ and let γ_1, γ_2 be continuous curves defined on $[0,1]$ with values in G such that $\gamma_1(0) = \gamma_2(0)$ is the center of D and $\gamma_1(1) = \gamma_2(1)$. Further suppose that there is a continuous mapping $H : [0,1] \times [0,1] \to G$ such that*

- $H(s,0) = \gamma_1(s)$ *for* $s \in [0,1]$,
- $H(s,1) = \gamma_2(s)$ *for* $s \in [0,1]$,
- $H(0,t) = \gamma_1(0)$ *for* $t \in [0,1]$,
- $H(1,t) = \gamma_1(1)$ *for* $t \in [0,1]$.

Then the function elements which are analytic continuation of (f, D) along γ_1 in G are the same as the function elements which are analytic continuation of (f, D) along γ_2 in G.

Theorem 6 (monodromy theorem). *Let Ω be a simply connected domain. Then any analytic multifunction in Ω which admits unrestricted continuation is singlevalued.*