Functional analysis 1 – introductory information

What is this course about and what is it good for

- As obvious from the title, this course is devoted to functional analysis. It is a wide area of mathematics, investigating among others infinite-dimensional vector spaces with an additional topological structure and continuous linear mappings.
- The content of the course is formed by four advanced areas of functional analysis:
  - Locally convex spaces and weak topologies – a generalization of normed spaces. Weak topologies are important e.g. for a deeper understanding of some properties of Banach spaces. Locally convex spaces provide a theory necessary to the study of some function spaces which are not normable (algebras of continuous, smooth or holomorphic functions, Schwartz space etc.). There exists a more general theory of topological vector spaces (covering in particular quasinormed spaces and p-Banach spaces) useful in the theory of function spaces. We will mention it only briefly.
  - Theory of distributions – generalized functions and measures. This is used namely in the study of partial differential equations.
  - Elements of vector integration – a generalization of the Lebesgue integral for functions with values in a Banach space. It is useful e.g. in the investigation of some partial differential equations.
  - Few facts on compact convex sets – on generating them using extreme points and integral representation. This is used in many areas of analysis, among others in differential equations. This is also a short introduction to a huge general theory.

Assumed knowledge

It is an advanced course of a Master program, to understand it one needs a nontrivial initial knowledge. Among others:

- Elements of functional analysis – normed linear spaces, Banach and Hilbert spaces, dual spaces, bounded linear operators, basic theorems of functional analysis. This knowledge is used throughout the course.
- Elements of general topology – topological spaces, base of a topology, neighborhood base, continuous mappings, basic topological constructions, compact spaces. This is necessary to understand the first and the fourth areas.
- Measure theory and Lebesgue integral – abstract measure, abstract Lebesgue integral, Lebesgue integral in $\mathbb{R}^n$. This is a key point for the second and the third area, but it is used in the remaining two as well.
- Real analysis – differential and integral calculus of one and several variables. This is important mainly in the second area.