3 test. 5. 1. 2021

$$\lim_{x\to\infty} \frac{x^\alpha}{a^x} = 0 \qquad a > 1$$

$$\lim_{n\to\infty} \frac{n^\alpha}{a^n} = 0 \qquad a > 1$$

$$\left[\lim_{n\to\infty} \frac{n^\alpha}{n!} = \lim_{n\to\infty} \frac{a^n}{n!} = \lim_{n\to\infty} \frac{n!}{n^n} = 0 \right]$$

$$\lim_{n\to\infty} \frac{3^n + 5^n}{n! + 6^n} = \lim_{n\to\infty} \frac{\frac{3^n}{n!}\to 0 + \frac{5^n}{n!}\to 0}{1 + \frac{6^n}{n!}\to 0} = \frac{0+0}{1+0} = 0$$

$$\lim_{n\to\infty} \frac{3^n + 5^n}{7^n + 6^n} = \qquad 2^{n^2} \qquad (2n)!$$

$$+\infty \cdot (+\infty - (+\infty))$$

$$\left[\lim_{n\to\infty} n\left(\sqrt{n+1} - \sqrt{n}\right) = +\infty \right.$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\lim_{n\to\infty} n \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty} \frac{n}{\sqrt{n+1}+\sqrt{n}}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}} \qquad \lim_{n\to\infty} \frac{}{\sqrt{n+1}+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{\frac{n}{\sqrt{n}}}{\sqrt{\frac{n+1}{n}} + 1} = \frac{+\infty}{1+1} = \underline{\underline{+\infty}}$$

$\frac{n}{\sqrt{n}} \sim \sqrt{n} \to +\infty$, $\quad \frac{n+1}{n} \to 1$

$$\lim_{n\to\infty} \sqrt[n]{3^n+5^n+1} = \qquad \underline{\sqrt[n]{5^n} = 5}$$

$n^4 \quad n^{430}$

$$5 = \sqrt[n]{5^n} \le \sqrt[n]{3^n+5^n+1} \le \sqrt[n]{5^n+5^n+5^n} = \sqrt[n]{3\,5^n} = 5\cdot\sqrt[n]{3}$$

$a_n \qquad c_n \qquad b_n$

$$\boxed{\lim_{n\to\infty} a_n = 5}$$

$$\boxed{\lim_{n\to\infty} b_n = 5\cdot\lim_{n\to\infty}\sqrt[n]{3} = 5}$$

$\lim_{n\to\infty} c_n = 5$

$$\lim_{n\to\infty}\sqrt[n]{n} = 1$$

s

$$\lim_{n\to\infty}\sqrt[n]{3} = \lim_{n\to\infty} 3^{\frac{1}{n}} \left[= \lim_{n\to\infty} e^{\frac{1}{n}\cdot\log 3} = e^0\right] = 1$$

$\frac{1}{n}\cdot\log 3 \to 0$

x

e^x spojitá v 0

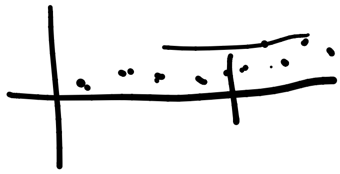
3^x spojitá v 0 $\quad 3^0 = 1$

$$\lim_{n\to\infty}\sqrt[n]{3^n+5^n+1} = 5$$

$$\lim_{n\to\infty}\sqrt[n]{5^n+n^{20}} =$$

$n^{20} \qquad n^{20}$

$$\frac{n^{20}}{5^n} \to 0 \qquad \frac{n^{20}}{5^n} \leq C$$



$$5^n \leq 5^n + n^{20} \leq \underline{(1+C)}\,5^n$$

- $$\lim_{n\to\infty} \underline{\frac{(2n)!}{(n!)^2}}$$

Podílové kritérium $\quad \lim\limits_{n\to\infty} a_n$

$a_n > 0$

- $$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L \begin{cases} <1 & \lim\limits_{n\to\infty} a_n = 0 \\ =1 & \text{nevíme} \\ \underline{>1} & \lim\limits_{n\to\infty} a_n = \underline{+\infty} \end{cases}$$

$\leftrightarrow \sqrt[n]{a_n}$

$$\boxed{\frac{a_{n+1}}{a_n} \to L < 1}$$

$$\frac{a_{n+1}}{a_n} > L+\varepsilon > 1 \qquad \boxed{\frac{a_{n+1}}{a_n}} < \underline{L+\varepsilon} \boxed{<1}$$

$$(L+\varepsilon)^n \to \infty \qquad a_{n+1} < (L+\varepsilon)\,a_n$$

- $$0 \leq \underset{k\to\infty}{a_{n+k}} < \underline{(L+\varepsilon)^k}_{\to 0}\, a_n$$
- $a_{n+k} \to 0$
  $a_n \to 0$

$\lim \quad 3^n$

- $\lim\limits_{n\to\infty} \frac{3^n}{n!} = 0$

$\boxed{\left(1+\frac{1}{n}\right)^n \to e}$ $\quad \frac{3^n}{n!} = a_n$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} = \frac{3}{n+1} \to 0 < 1$$

- $\lim\limits_{n\to\infty} \frac{n!}{n^n} = 0$

- $\lim\limits_{n\to\infty} \frac{(2n)!}{(n!)^2} = +\infty$

$\frac{(2n)!}{(n!)^2} = a_n$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(2n+2)!}{[(n+1)!]^2}}{\frac{(2n)!}{(n!)^2}} = \lim_{n\to\infty} \frac{(2n+2)(2n+1)}{(n+1)^2}$$

$$= \lim_{n\to\infty} \frac{4n^2+6n+2}{\underline{n^2+2n+1}} = \lim_{n\to\infty} \frac{4+\frac{6}{n}+\frac{2}{n^2}}{1+\frac{2}{n}+\frac{1}{n^2}} \quad \to 0$$

$$= \underline{\underline{4}} > 1 \to \lim_{n\to\infty} a_n = +\infty$$

$\lim\limits_{n\to\infty} \frac{n!}{2^{(n^2)}} = ?$

$$\lim_{n\to\infty} \ldots 2^{(n^2)} = \ldots$$

(7) $\lim_{n\to\infty} a_n = ?$ $\quad a_{n+1} = \sqrt{a_n+2} \quad a_1 = \sqrt{2}$

pokud $a_n \to L$ potom $L > 0$ (≥ 1) •

$\underline{L \in \mathbb{R}}$ $\quad L = \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{a_n+2} = \sqrt{L+2}$

$a_n + 2 \to L+2$

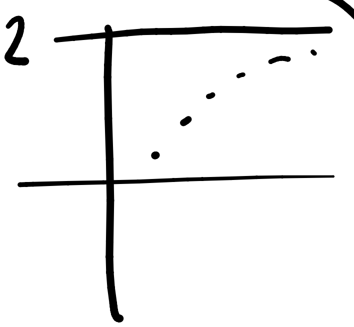
• $L = \sqrt{L+2} \iff L^2 = L+2 \; \langle \; 2, \; \cancel{-1}$ •

$L > 0$

$\underline{L = +\infty}$ ↯ $\quad \underset{+\infty}{a_{n+1}} = \sqrt{\underset{+\infty}{a_n+2}}$

[• $a_n \overset{?}{\leq} 2$ • $\{a_n\}$ rostoucí (neklesající) •

$\lim_{n\to\infty} a_n = L \in \mathbb{R}$



[$\boxed{\bullet \;\; a_n \leq 2}$ • $a_1 = \sqrt{2} < 2$

$\lim_{n\to\infty} a_n \in \mathbb{R}$

• $a_n \leq 2 \implies a_{n+1} \leq 2$

$< a_{n+1} = \sqrt{a_n+2} \leq \sqrt{2+2} = 2$

$<$ $a_{n+1} = \sqrt{a_n + 2} \le \sqrt{2+2} = 2$

$\lim\limits_{n\to\infty} a_n \in \mathbb{R}$

$\downarrow$

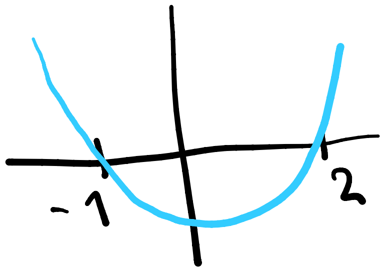
$= 2$

$\boxed{\bullet\ \{a_n\} \text{ neklesající}}$

$a_n \le a_{n+1} = \sqrt{a_n + 2}$ $\qquad$ $a_n \ge 0$

$a_n^2 \le a_n + 2 \qquad \bullet\, a_n^2 - a_n - 2 \le 0\, \bullet$

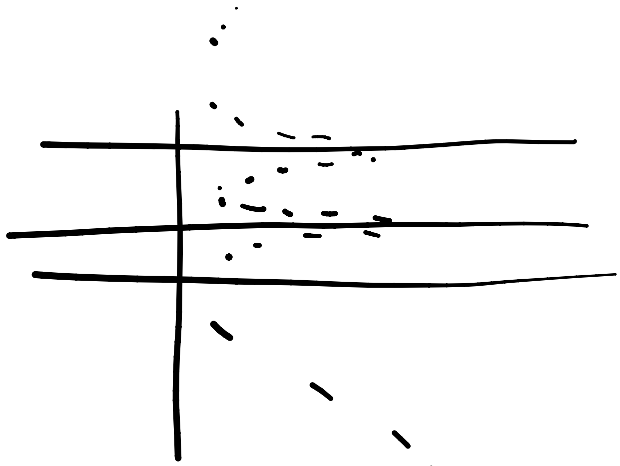
$a_n \le a_{n+1}$ ✓

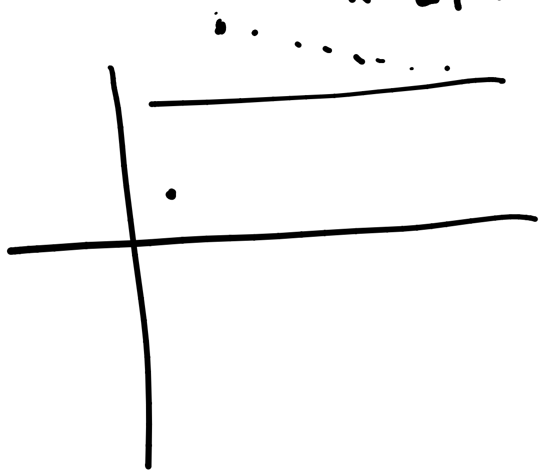


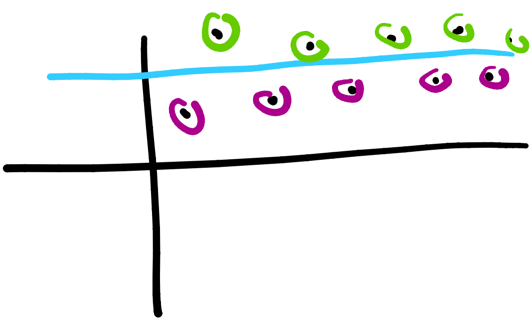
$x^2 - x - 2$

$\le 0 \quad x \in [-1, 2]$

$x \in [0, 2]$







- $a_n = (-1)^n \cdot \underbrace{\left(1 + \frac{1}{n}\right)}_{\to 0} \longrightarrow 1 \ne 0$

- $a_n = (-1)^n \underbrace{\left(1 + \frac{1}{n}\right)}_{} \to 1 \neq 0$

- $a_{2n} = 1 + \underbrace{\frac{1}{2n}}_{\nearrow 0} \to 1$
- $a_{2n-1} = -\left(1 + \underbrace{\frac{1}{2n-1}}_{\searrow 0}\right) \to -1$

$\Big\}$ $\lim\limits_{n\to\infty} a_n$ ~~ne~~ existuje

$$\lim_{n\to\infty} (-1)^n \overbrace{\left(\sqrt{n+1} - \sqrt{n}\right)}^{a_n} =$$

- $\lim\limits_{n\to\infty} \sqrt{n+1} - \sqrt{n} = \lim\limits_{n\to\infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \lim\limits_{n\to\infty} \frac{1}{\underbrace{\sqrt{n+1}}_{\to +\infty} + \underbrace{\sqrt{n}}_{\to +\infty}} = 0$

$(a_n > 0)$

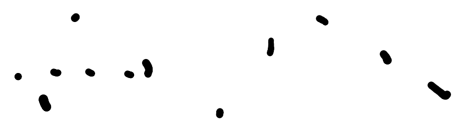
$$\underbrace{-a_n}_{\alpha_n} \leq \underbrace{(-1)^n a_n}_{\to 0} \leq \underbrace{a_n}_{\beta_n}$$

$\beta_n \to 0$

$\alpha_n = -\beta_n \to 0$

- $b_n = (-1)^n a_n \qquad a_n > 0 \qquad \lim\limits_{n\to\infty} b_n$ existuje $\Leftarrow \lim\limits_{n\to\infty} a_n = 0$

$$a_n \begin{cases} \cos(\pi n) = (-1)^{n+1} \\ \{\cos(\frac{\pi}{2} n)\} = \{0, -1, 0, +1, 0, \dots\} \\ \{\cos(\frac{\pi}{10} n)\} = \{ \\ \{\cos(n)\} = \end{cases}$$



$\left(\{\cos(n)\} = \right.$ 

$a_n \cdot b_n \to \ ? \qquad b_n \to 0 \qquad (b_n > 0)$

$$\limsup_{n\to\infty} a_n = \begin{cases} \lim\limits_{n\to\infty} \sup\{a_k : k \geq n\} & \{a_n\} \text{ omezená shora} \\ +\infty & \text{jinak} \end{cases}$$

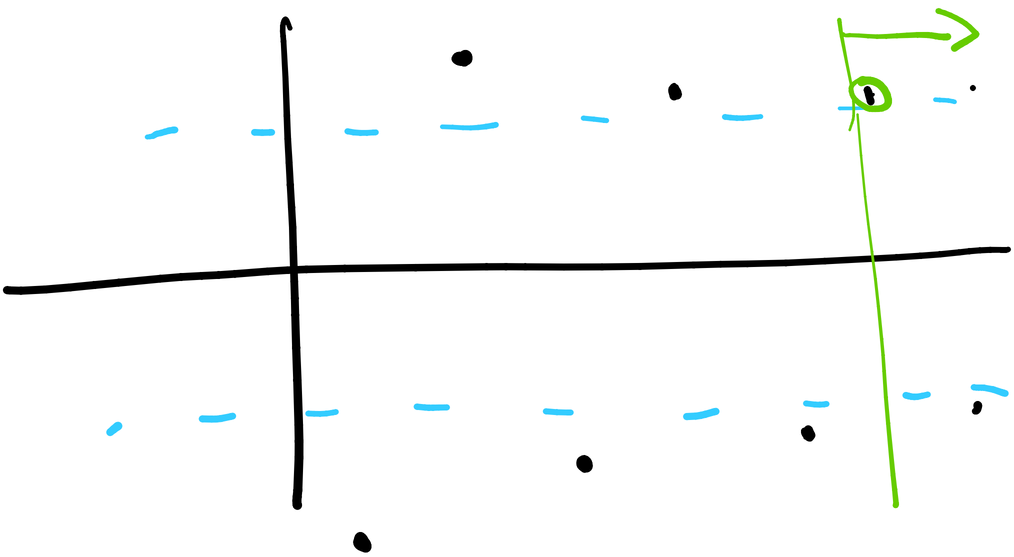
$n > 300$

$$a_n = (-1)^n\left(1+\frac{1}{n}\right) \quad \begin{matrix} 1 \\ \frac{3}{2} \end{matrix}$$

$-(1+1),\ 1+\frac{1}{2},\ -(1+\frac{1}{3}),\ 1+\frac{1}{4} \qquad 1+\frac{1}{6}$

n sudé $\qquad n=2 \qquad n=4 \qquad n=6$

$$\sup\{a_k : k \geq n\} = 1+\frac{1}{n} \to 1$$

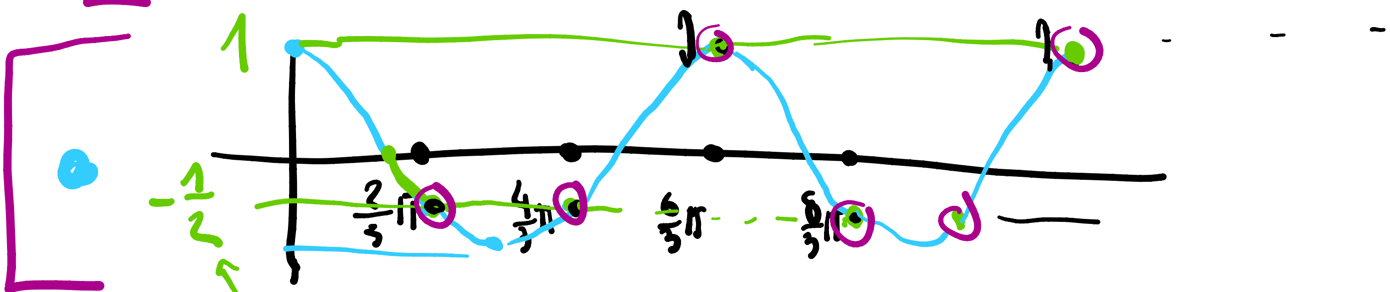


$\limsup a_n = 1$

$\limsup a_n = 1$

$\limsup\limits_{n\to\infty} \frac{n-1}{n+1} \cos \frac{2}{3} n\pi$ $\quad$ ($\liminf = ?$)

$\limsup\limits_{n\to\infty} n\left(2+(-1)^n\right) \geq n$ $\quad$ ($\liminf = ?$)



$-\sin\left(\frac{\pi}{6}\right)$

$\frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$ $\qquad n \to \infty$

$\limsup a_n = 1$

$\inf = -\frac{1}{2}$

$\lim\limits_{n\to\infty} \frac{n-1}{n+1} = 1$

$n(1)$

$n([illegible])$

$n(3)$