Cvičení 5.5.

středa 5. května 2021 13:00

$$\sum \frac{(-1)^n x^{n+3}}{\cancel{(n+3)}\, n!} \qquad x^{n+2} \to \qquad \int x^2 e^{-x}$$

$f : \mathbb{R}^d \to \mathbb{R}$

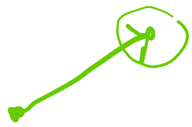
$$\nabla f(x) = \underbrace{\left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_d} \right)}_{d}$$

$F : \mathbb{R}^d \to \mathbb{R}^n \qquad F = (F_1, \dots, F_n) \qquad F_n : \mathbb{R}^d \to \mathbb{R}$

$$J_F = \begin{pmatrix} \nabla F_1 \\ \vdots \\ \nabla F_n \end{pmatrix} = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \dots & \frac{\partial F_1}{\partial x_d} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial x_1} & \dots & \frac{\partial F_n}{\partial x_d} \end{pmatrix}$$

tot. diferenciál

$$\lim_{\mathbb{R}^d \ni h \to \vec{0}} \frac{f(x+h) - f(x) - L \cdot h}{\|h\|} = 0$$

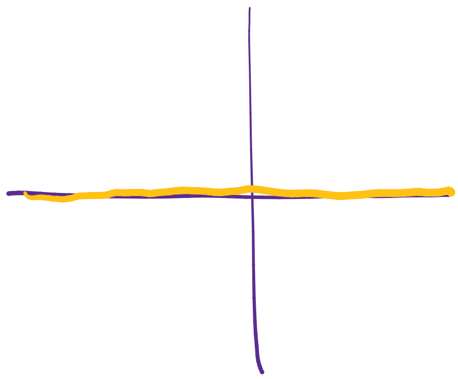


$$L(h) = \nabla f(x) \cdot h$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x) - f'(x) \cdot h}{h} = 0$$

(6) MA!

$$f(x,y) = \begin{cases} \dfrac{x^5 + y^5}{x^4 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$



tot. diferencovatelná ve všech bodech $\mathbb{R}^2$?

Mimo $(0,0)$: pokud má f parciální derivace spojité v (x,y), potom existuje tot. diferenciál f v (x,y) $\left[f'(x,y),\ D f(x,y),\ D_f(x,y) \right]$ ✓

$$f'(x,y)(h) = \nabla f \cdot h$$

V $(0,0)$

$$f(x,0) = \begin{cases} \frac{x^5}{x^4} = x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$g(x) = f(x,0) = x \qquad \frac{\partial f}{\partial x}(0,0) = g'(0) = 1$$

$$h(y) = f(0,y) = \frac{y^4}{y^2} = y^2 \qquad \frac{\partial f}{\partial y}(0,0) = h'(0) = 2 \cdot 0 = 0$$

$$h'(y) = 2y$$

$$\nabla f(0,0) = (1,0)$$

$$\lim_{(a,b)\to 0} \frac{f(0+a,0+b) - f(0,0) - \nabla f(0,0)\cdot(a,b)}{\sqrt{a^2+b^2}} = 0$$

$a^5 + b^4 \quad 0 \quad (1,0)\cdot(a,b) \qquad \frac{x^5+y^4}{\ldots}$

$$\lim_{(a,b)\to 0} \frac{\frac{a^5+b^4}{a^4+b^2} - 0 - (1,0)\cdot(a,b)}{\sqrt{a^2+b^2}} \qquad \frac{x^5+y^4}{x^4+y^2}$$

$$\lim_{(a,b)\to 0} \frac{\frac{a^5+b^4}{a^4+b^2} - a}{\sqrt{a^2+b^2}} \quad f(a,b)$$

$$f(a,a) = \left(\frac{a^5+a^4}{a^4+a^2} - a\right)\frac{1}{\sqrt{2}|a|}$$

$$= \frac{a^5+a^4-a^5-a^3}{\sqrt{2}|a|(a^4+a^2)} = \frac{a^4-a^3}{\sqrt{2}|a|(a^4+a^2)} \to$$

$$f(r\cos\varphi, r\sin\varphi) = \frac{\frac{r^5\cos^5\varphi+r^4\sin^4\varphi}{r^4\cos^4\varphi+r^2\sin^2\varphi} - r\cos\varphi}{r}$$

$$= \frac{\frac{r^3\cos^5\varphi+r^2\sin^4\varphi}{r^2\cos^4\varphi+\sin^2\varphi} - r\cos\varphi}{r}$$

$$= \frac{r^2\cos^5\varphi+r\sin^4\varphi}{r^2\cos^4\varphi+\sin^2\varphi} - \cos\varphi$$

neexistuje

(6) $f(x,y) = \begin{cases} \dfrac{\cancel{x^5}+y^5}{\cancel{y^2}+y^2} & (x,y) \neq (0,0) \\ \underline{\underline{0}} & (x,y) = (0,0) \end{cases}$

$g(x) = f(x,0) = x^3 \rightarrow g'(x) = 3x^2 \quad g'(0) = 0$

$\quad \| \quad \frac{\partial f}{\partial x}(0,0)$

$f(0,y) = y^2 \rightarrow (y^2)' = 2y \qquad \frac{\partial f}{\partial y}(0,0) = 0$

$\nabla f(0,0) = \underline{\underline{(0,0)}}$

$$\lim_{(a,b)\to 0} \frac{f(0+a,0+b) - f(0,0) - \nabla f(0,0)\cdot(a,b)}{\sqrt{a^2+b^2}} = 0$$

$$= \lim_{(a,b)\to 0} \frac{\frac{a^5+b^4}{a^2+b^2} - \cancel{0} - \cancel{\overset{\| \, 0}{(0,0)\cdot(a,b)}}}{\sqrt{a^2+b^2}}$$

$$= \lim \frac{a^5+b^4}{(a^2+b^2)^{\frac{3}{2}}} \rightarrow \begin{array}{l} a = r\cos\varphi \\ b = r\sin\varphi \end{array} \qquad r^2 \leq r \quad 0<r<1$$

$$= \lim_{(a,b)\to 0} \frac{\quad}{(a^2+b^2)^{\frac{3}{2}}} \qquad b = r\sin\varphi \qquad \begin{array}{c} r^2 \le r \\ 0<r<1 \end{array}$$

r^2

$$\left| \frac{r^5\cos^5\varphi + r^5\sin^4\varphi}{r^3} - 0 \right| = r^2|\cos^5\varphi| + r\sin^4\varphi \le 2r \xrightarrow{r\to 0^+} 0$$

≤ 1, ≤ 1

f má tot. diferenciál v bodě $(0,0)$.

Mimo $(0,0)$ má f spojité parciální derivace

$\Rightarrow$ tot. diferenciál

(8) $f(x,y) = \sqrt[3]{x^3+y^3}$

Má f tot. diferenciál ve všech bodě $(0,0)$.

$$g(x) = f(x,0) = x \qquad (x)' = 1 \qquad \frac{\partial f}{\partial x}(0,0) = 1$$

$$= f(0,y) = y \qquad (y)' = 1 \qquad \frac{\partial f}{\partial y}(0,0) = 1$$

$$\nabla f(0,0) = (1,1)$$

$$\lim_{(a,b)\to 0} \frac{\sqrt[3]{a^3+b^3} - \overset{0}{f(0,0)} - (a,b)\cdot(1,1)}{\sqrt{a^2+b^2}}$$

$(a,b)\to 0$ $\qquad\qquad \sqrt{a^2+b^2}$

$$\lim_{(a,b)\to 0} \frac{\sqrt[3]{a^3+b^3} - a - b}{\sqrt{a^2+b^2}}$$

$$\lim_{a\to 0} f(a,a) = \lim_{a\to 0} \frac{\sqrt[3]{2a^3} - 2a}{\sqrt{2a^2}}$$

$$= \lim_{a\to 0} \frac{\sqrt[3]{2}\cdot a - 2a}{\sqrt{2}\cdot \cancel{|}a\cancel{|}}$$

<u>neexistuje</u>

$$\lim_{a\to 0\pm} \frac{\sqrt[3]{2}\,a - 2a}{\sqrt{2}\,a} = \pm\frac{\sqrt[3]{2}-2}{\sqrt{2}} \neq 0.$$