$$\int_0^{\log 4} \sqrt{e^x-1}\,dx$$

$$\int \sqrt{e^x-1}\,dx = \int e^x \frac{\sqrt{e^x-1}}{e^x}\,dx = \left| \begin{array}{l} t = e^x \\ dt = e^x dx \end{array} \right|$$

$t = e^x - 1$?

$\sqrt{t-1}$

$$= \int \frac{\sqrt{t-1}}{t}\,dt = \left| \begin{array}{l} u = \sqrt{t-1} \\ t = u^2+1 \\ dt = 2u\,du \end{array} \right|$$

$$R\left(x, \sqrt[s]{\frac{ax+b}{cx+d}}\right)$$

t

$$= \int \frac{u}{u^2+1}\cdot 2u\,du = 2\int \frac{u^2}{u^2+1}\,du$$

$$= 2\int \frac{u^2+\overbrace{1-1}^{=0}}{u^2+1}\,du = 2\int 1\,du - 2\int \frac{1}{u^2+1}\,du$$

$$\overset{c}{=} 2u - 2\operatorname{arctg} u = 2\sqrt{t-1} - 2\operatorname{arctg}\sqrt{t-1}$$

$$= 2\sqrt{e^x-1} - 2\operatorname{arctg}\sqrt{e^x-1}$$

$$\int_0^{\log 4} \sqrt{e^x-1}\,dx = \left[2\sqrt{e^x-1} - 2\operatorname{arctg}\sqrt{e^x-1} \right]_0^{\log 4}$$

$$\int_0^{1} \sqrt{e^x-1}\,dx = \Big[2\sqrt{e^x-1} - 2\arctg\sqrt{e^x-1}\Big]_0$$

$$e^0-1=0$$

$$\Big[F(x)\Big]_a^b = \lim_{x\to b^-} F(x) - \lim_{x\to a^+} F(x) \qquad e^{\log 4} = 4$$

$$= 2\sqrt{4-1} - 2\arctg\sqrt{4-1} - 0$$

$$= \underline{\underline{2\sqrt{3} - 2\frac{\pi}{3}}}$$

$$\int_0^{\log 4} \sqrt{e^x-1}\,dx = \left| \begin{array}{ll} t = e^x & dt = e^x dx \\ x=0 \to & t = \boxed{1} \\ x = \log 4 \to & t = \boxed{4} \end{array} \right|$$

$$= \int_1^4 \frac{\sqrt{t-1}}{t}\,dt = \left| \begin{array}{lll} u=\sqrt{t-1} & t = u^2+1 & dt = 2u\,du \\ t=1 & \to u=0 & \\ t=4 & \to u=\sqrt{3} & \end{array} \right|$$

$$= 2\int_0^{\sqrt{3}} \frac{u^2}{u^2+1}\,du = 2\int_0^{\sqrt{3}} 1\,du - 2\int_0^{\sqrt{3}} \frac{1}{u^2+1}\,du$$

$$\arctg\sqrt{3}$$

$$= 2\Big[u\Big]_0^{\sqrt{3}} - 2\Big[\arctg u\Big]_0^{\sqrt{3}} = \underline{\underline{2\sqrt{3} - 2\frac{\pi}{3}}}$$

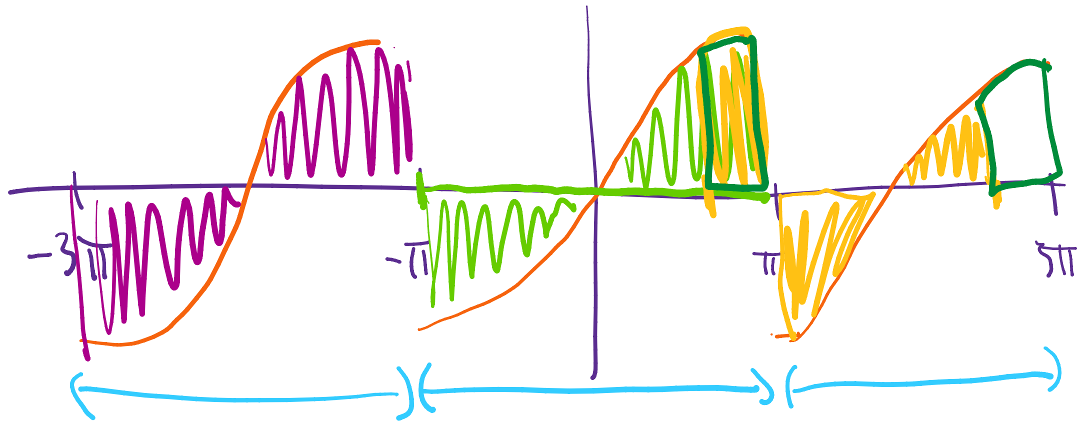
$$\bullet \int_0^{4\pi} \frac{1}{1+\sin^2 x} = ?$$

$$\int_0^{2\pi} \sin^2 x \, dx = I$$

$$\sin^2 x + \cos^2 x = 1$$

$$\int_0^{2\pi} \cos^2 x \, dx = I$$

$$2I = \int_0^{2\pi} 1 = 2\pi$$



$$\int_0^{4\pi} \frac{1}{1+\sin^2 x} dx = 4 \int_0^{\pi} \frac{1}{1+\sin^2 x} dx$$

$(-\infty, \infty)$

$\bullet$ $t = \operatorname{tg} x$ $\qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi \quad k \in \mathbb{Z}$

• $t = \operatorname{tg} x \qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi \qquad k \in \mathbb{Z}$

$$4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+\sin^2 x}\,dx = \left|\begin{array}{ll} t = \operatorname{tg} x & dx = \frac{1}{1+t^2}\,dt \\ \sin^2 x = \frac{t^2}{1+t^2} & x = -\frac{\pi}{2} \to t = -\infty \\ & x = \frac{\pi}{2} \to t = +\infty \end{array}\right|$$

$$\lim_{x \to -\frac{\pi}{2}+} \operatorname{tg} x = -\infty \qquad \lim_{x \to \frac{\pi}{2}-} \operatorname{tg} x = +\infty$$

$$4\int_{-\infty}^{\infty} \frac{1}{1+\frac{t^2}{1+t^2}} \cdot \frac{1}{1+t^2}\,dt = 4\int_{-\infty}^{\infty} \frac{1}{(\sqrt{2}t)^2+1}\,dt$$

$$= \frac{4}{\sqrt{2}}\int_{-\infty}^{\infty} \frac{1\,\sqrt{2}}{(\sqrt{2}t)^2+1}\,dt = \left|\begin{array}{ll} u = \sqrt{2}t & du = \sqrt{2}\,dt \\ t = \pm\infty \to u = \pm\infty & \end{array}\right|$$

$$= \frac{4}{\sqrt{2}}\int_{-\infty}^{\infty} \frac{1}{u^2+1}\,du = \frac{4}{\sqrt{2}}\Big[\operatorname{arctg} u\Big]_{-\infty \,(\to -\frac{\pi}{2})}^{+\infty \,(\to \frac{\pi}{2})} = \frac{4}{\sqrt{2}}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

$$= \frac{4}{\sqrt{2}}\pi$$

$$\int_0^1 x^a\,dx = \begin{cases} \text{konverguje} & a > -1 \\ \text{diverguje} & a \leq -1 \end{cases}$$

$$\int_1^\infty x^a\,dx = \begin{cases} \text{konverguje} & a < -1 \\ \text{diverguje} & a \geq -1 \end{cases}$$

$a = -1$

$\downarrow$

$\frac{1}{x}$

$$\int_0^{\frac{1}{2}} \frac{1}{x\,|\log x|^a}\,dx = \begin{cases} \text{konverguje} \\ \text{diverguje} \end{cases}$$

$$\int_2^\infty \frac{1}{x \log^a x}\,dx = \begin{cases} \text{konverguje} \\ \text{diverguje} \end{cases}$$

$$\int \frac{1}{x \log^a x}\,dx = \left| \begin{matrix} t = \log x \\ dt = \frac{1}{x}\,dx \end{matrix} \right| = \int \frac{1}{t^a}$$

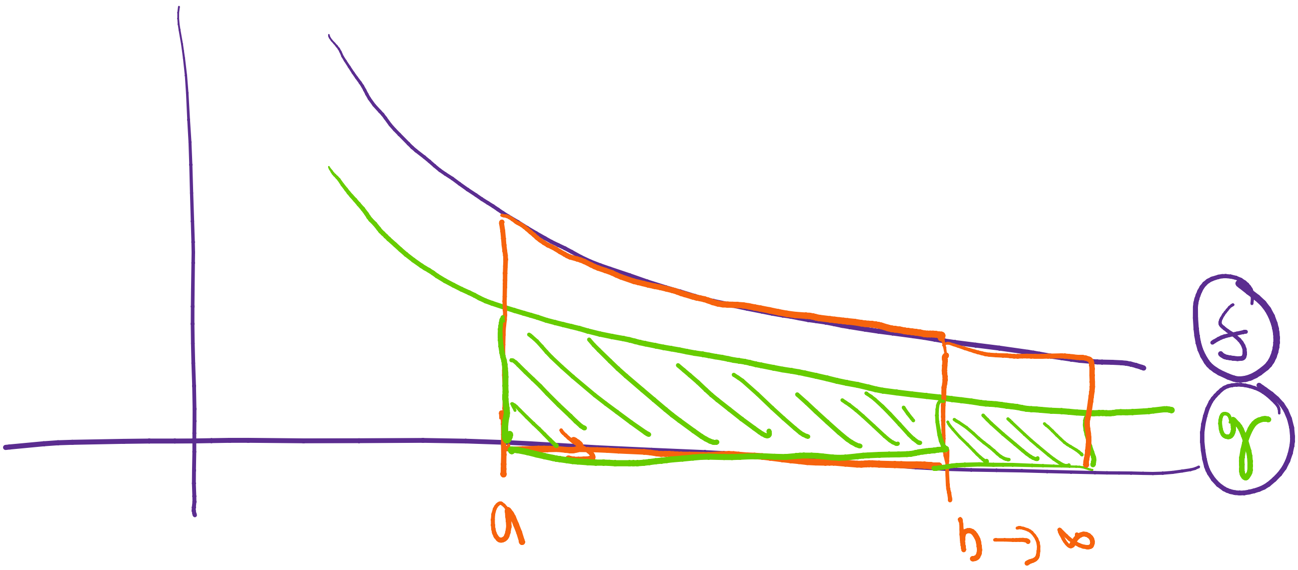
$$\int_0^{\frac{1}{2}} \frac{1}{x\,|\log x|^a}\,dx = \left| \begin{matrix} t = \log x & dt = \frac{1}{x}\,dx \\ x = 0 & t = -\infty \end{matrix} \right|$$

$$\int_0 \frac{}{x|\log x|^a}\,dx = \left| \begin{matrix} x=0 & t=-\infty \\ x=\frac{1}{2} & t=\log\frac{1}{2} \end{matrix} \right|$$

$$= \int_{-\infty}^{\log\frac{1}{2}} \frac{1}{|t|^a} = \int_{-\log\frac{1}{2}}^{\infty} \frac{1}{t^a} = \begin{cases} \text{konvergują} & a>1 \\ \text{divergują} & a\le 1 \end{cases}$$

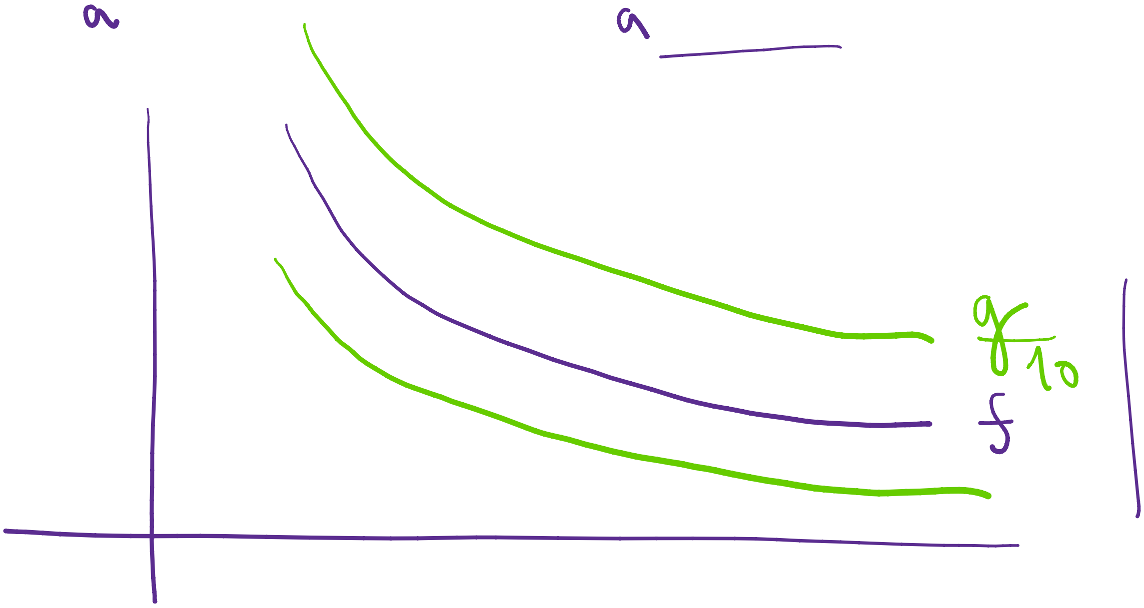
$$\int_0^\infty \frac{x^{\frac{3}{2}}}{1+x^2}\,dx$$

$$\int_0^\infty \frac{\operatorname{arctg} x}{x^{\frac{3}{2}}}\,dx$$



$$\int_a^\infty f \underline{< \infty} \implies \int_a^\infty g \underline{< \infty}.$$

$$\int_a^\infty g = \infty \implies \int_a^\infty f = \infty$$



- $$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \underline{C} \in (0,\infty)$$

$$\boxed{f \sim C\cdot g \qquad x \to \infty}$$

$$\frac{C}{2} g \le f \le (C+1)\, g$$

$$\int_a^\infty f \text{ konvergira} \iff \int_a^\infty g \text{ konvergira}$$

a a

	~ 0		$\sim \infty$		
$\frac{x^{\frac{3}{2}}}{1+x^2}$	$x^{\frac{3}{2}}$	K	$x^{-\frac{1}{2}}$	D	D
$\frac{\underset{\sim c}{\arctan} \underset{\sim x}{x}}{x^{\frac{3}{2}}}$	$x^{-\frac{1}{2}}$	K	$x^{-\frac{3}{2}}$	K	K

$x^{\frac{3}{2}} \int$

$x^{\frac{3}{2}} \int$

$$\lim_{x\to 0} \frac{\arctan x}{x} = 1$$

$a = \frac{3}{2}$

$$\lim_{x\to 0+} \frac{\frac{x^{\frac{3}{2}}}{1+x^2}}{x^a} = \lim_{x\to 0+} \frac{x^{\frac{3}{2}-a}}{1+x^2} = 1$$

$a = -\frac{1}{2}$

$$\lim_{x\to\infty} \frac{\frac{x^{\frac{3}{2}}}{1+x^2}}{x^a} = \lim_{x\to\infty} \frac{x^{\frac{3}{2}-a}}{1+x^2} = \lim_{x\to\infty} \frac{x^{\frac{3}{2}-a-2}}{\frac{1}{x^2}+1}$$

∞

$$\frac{x^{\frac{3}{2}}}{1+x^2} \sim \frac{x^{\frac{3}{2}}}{x^2} \sim x^{-\frac{1}{2}}$$

$x^{\frac{3}{2}} \sim x^{\frac{3}{2}}$, $1+x^2 \sim x^2$

$\int_1^\infty$

$$\int_2 \frac{1}{x^7 \log x}$$

$$\frac{\log x}{x} \searrow 0 \qquad \frac{\cancel{\log x}^{\,x\to\infty}}{x^{7\,6}}\searrow \infty \qquad \cancel{x} \quad x^{9}$$

$\int_0^1 \arccos x \, dx$	$\int_0^{\frac{\pi}{2}} \tan x \, dx$ $\quad t = \cos x$

$$\frac{\sin x}{\cos x}$$

$$\int 1 \cdot \arccos x \, dx = PP \to$$

$$\int_0^1 1 \cdot \arccos x \, dx = \left| \begin{array}{ll} f = \arccos x & f' = \frac{-1}{\sqrt{1-x^2}} \\ g' = 1 & g = x \end{array} \right|$$

$$\left[\; \right]^{1} \int^{1} -2(x)$$

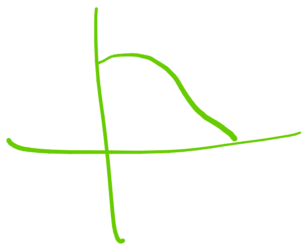
$$= \left[\overset{0}{\overset{\shortparallel}{x}}\, \overset{\shortparallel}{\arccos x}\right]_0^1 + -\frac{1}{2}\int_0^1 \frac{-2x}{\sqrt{1-x^2}}\,dx$$

$$= \left| \begin{array}{lll} t = 1-x^2 & & dt = -2x\,dx \\ x = 0 & \to & t = 1 \\ x = 1 & \to & t = 0 \end{array} \right|$$

$$= -\frac{1}{2}\int_1^0 \frac{1}{\sqrt{t}}\,dt = \frac{1}{2}\int_0^1 \frac{1}{\sqrt{t}}\,dt \leftarrow t^{-\frac{1}{2}}$$

$$\int x^\alpha = \frac{x^{\alpha+1}}{\alpha+1}$$

$$= \cancel{\frac{1}{2}}\left[\frac{t^{\frac{1}{2}}}{\cancel{\frac{1}{2}}}\right]_0^1 = \underline{\underline{1}}$$



$$\int_0^{\frac{\pi}{2}} tg\,x\,dx = -\int_0^{\frac{\pi}{2}} \frac{-\sin x}{\cos x}\,dx = \left| \begin{array}{ll} t = \cos x & dt = -\sin x\,dx \\ x = 0 & t = 1 \\ x = \frac{\pi}{2} & t = 0 \end{array} \right|$$

$$-\int_1^0 \frac{1}{t}\,dt = \int_0^1 \frac{1}{t}\,dt = \left[\log t\right]_0^1 = 0 - (-\infty) = \underline{\underline{\infty}}$$

$$-\lim \log t = -(-\infty)$$

$$-\lim_{t \to 0+} \log t = -(-\infty)$$

$t = \operatorname{tg} x$

$F(\quad)$

$\sin^2 x, \cos^2 x, \sin x \cos x$