Cvičení 5.1.

úterý 5. ledna 2021 14:09

Taylor

$1 \; , \; 0$

• $$\left[\frac{\sin x}{x} = 1 - \frac{x^2}{6} + \frac{x^4}{120}\right]$$

$$\sin x = \underbrace{x - \frac{x^3}{6} + \frac{x^5}{120}} + o(x^5)$$

$$\frac{\sin x}{\cos x}$$

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$

$$\cancel{x} - \frac{x^3}{6} + \frac{x^5}{120} : \underline{1 - \frac{x^2}{2} + \frac{x^4}{24}} = \underline{x} + \underline{\frac{x^3}{3}} + \underline{\frac{2x^5}{15}}$$

$$-\left(\cancel{x} - \frac{x^3}{2} + \frac{x^5}{24}\right)$$

$$\cancel{\frac{x^3}{3}} - \frac{x^5}{30}$$

$$-\left(\frac{x^3}{3} - \frac{x^5}{6}\right)$$

$$\cancel{\frac{2x^5}{15}}$$

$$-\frac{x^5}{30} + \frac{x^5}{6} = \frac{2x^5}{15}$$

$$T^5_{0,\operatorname{tg} x}(x) = x + \frac{x^3}{3} + \frac{2x^5}{15}$$

Neurčité koeficienty

$$\frac{\sin x}{\cos x} = \frac{x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)}{1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)} = \alpha x + \beta x^3 + \gamma x^5 + o(x^5)$$

$$x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5) = \left(1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right)\left(\alpha x + \beta x^3 + \gamma x^5 + o(x^5)\right)$$

$$1\cdot x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5) = \alpha x + x^3\left(\beta - \frac{\alpha}{2}\right) + x^5\left(\frac{\alpha}{24} - \frac{\beta}{2} + \gamma\right)$$

$$\alpha = 1$$

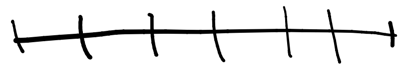
$$-\frac{1}{6} = \beta - \frac{\alpha}{2} = \beta - \frac{1}{2} \Rightarrow \beta = \frac{1}{3}$$

$$\frac{1}{120} = \frac{\alpha}{24} - \frac{\beta}{2} + \gamma = \frac{1}{24} - \frac{1}{6} + \gamma$$

$$\gamma = \frac{1}{120} - \frac{1}{24} + \frac{1}{6} = \frac{1 - 5 + 20}{120} = \frac{16}{120} = \frac{2}{15}$$

$$\operatorname{tg} x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + o(x^5)$$

$$(R)\int_0^1 x^2$$



ó

$$[F(x)]_{x=a}^{x=b}$$

$$(N)\int_a^b f = \overbrace{\lim_{x\to b-} F(x) - \lim_{x\to a+} F(x)}^{\text{přírůstek } F \text{ od } a \text{ do } b}, \bullet$$

kde F je nějaká primitivní funkce k f

$$(N)\int_a^b f = [F]_a^b$$

(1) • $\displaystyle\int_0^{\log 4} \sqrt{e^x-1}\,dx$

$$\int \sqrt{e^x-1}\,dx = \int \frac{\sqrt{e^x-1}}{e^x}\cdot e^x dx = \left| \begin{matrix} t = e^x \\ dt = e^x dx \end{matrix} \right|$$

$$= \underline{\int \frac{\sqrt{t-1}}{t}\,dt} = \underline{\underline{2\int \frac{u^2 \;{\scriptstyle +1-1}}{u^2+1}\,du}}$$

$\boxed{u} = \boxed{\sqrt{t-1}}$

$u^2 = t-1$

$$= 2\int \frac{u^2+1}{u^2+1} - \frac{1}{u^2+1}\,du$$

$$= 2\int 1\,du - 2\int \frac{1}{\ldots}\,du$$

$u^2 = t-1$

$\boxed{t = u^2+1}$

$\boxed{dt = 2u\,du}$

$$= 2\int 1\,du - 2\int \frac{1}{u^2+1}\,du$$

$$= 2u - 2\,\mathrm{arctg}\,u$$

$$= 2\sqrt{e^x-1} - 2\,\mathrm{arctg}\sqrt{e^x-1}$$

$u = \sqrt{t-1}$

$t = e^x$

$$\int_0^{\log 4} \sqrt{e^x-1}\,dx = \Big[2\sqrt{e^x-1} - 2\,\mathrm{arctg}\sqrt{e^x-1}\Big]_{x=0}^{x=\log 4}$$

$\lim\limits_{x\to b^-} F(x)$

$$= 2\sqrt{4-1} - 2\,\mathrm{arctg}\sqrt{4-1}$$

$\lim\limits_{x\to a^+} F(x)$

$$-\left(2\sqrt{1-1} - 2\,\mathrm{arctg}\sqrt{1-1}\right) = 0$$

$$= 2\sqrt{3} - 2\,\mathrm{arctg}\sqrt{3}$$

$$\left(= 2\sqrt{3} - 2\,\frac{\pi}{3}\right)$$

$$\int_0^{\log 4} \sqrt{e^x-1}\,dx = \left|\begin{array}{l} t = e^x \\ dt = e^x dx \\ \boxed{\begin{array}{l} x=0 \to t=1 \\ x=\log 4 \to t=4 \end{array}} \end{array}\right| = \int_1^4 \frac{\sqrt{t-1}}{t}\,dt$$

$u = \sqrt{t-1}$

$$2\int^{\sqrt{3}} \frac{u^2}{\ldots}$$

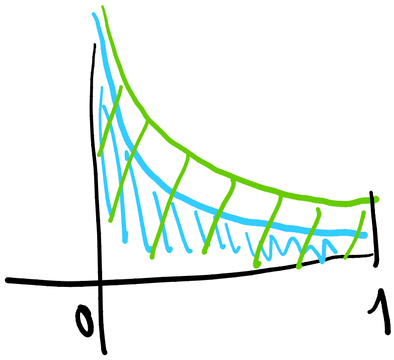
$$= \left| \begin{array}{ll} u=\sqrt{t-1} & \\ t=u^2+1 & \\ t=1 & u=0 \\ t=4 & u=\sqrt{3} \end{array} \right| = 2\int_0^{\sqrt{3}} \frac{u^2}{u^2+1}\,du$$

$$= \left[2u - 2\operatorname{arctg} u\right]_0^{\sqrt{3}} = \underline{\underline{2\sqrt{3} - 2\operatorname{arctg}\sqrt{3}}} \;(-0)$$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \int_0^1 x^{-\frac{1}{2}}\,dx = \left[2\cdot x^{\frac{1}{2}}\right]_0^1 = 2-0=2$$

$$/\!\wedge$$

$$\int_0^1 \frac{1}{x^2}\,dx = \int_0^1 x^{-2} = \left[-x^{-1}\right]_0^1 = -1-(-\infty) = +\infty$$



$$\underline{\underline{\frac{1}{\sqrt{x}}}} < \underline{\underline{\frac{1}{x^2}}}$$

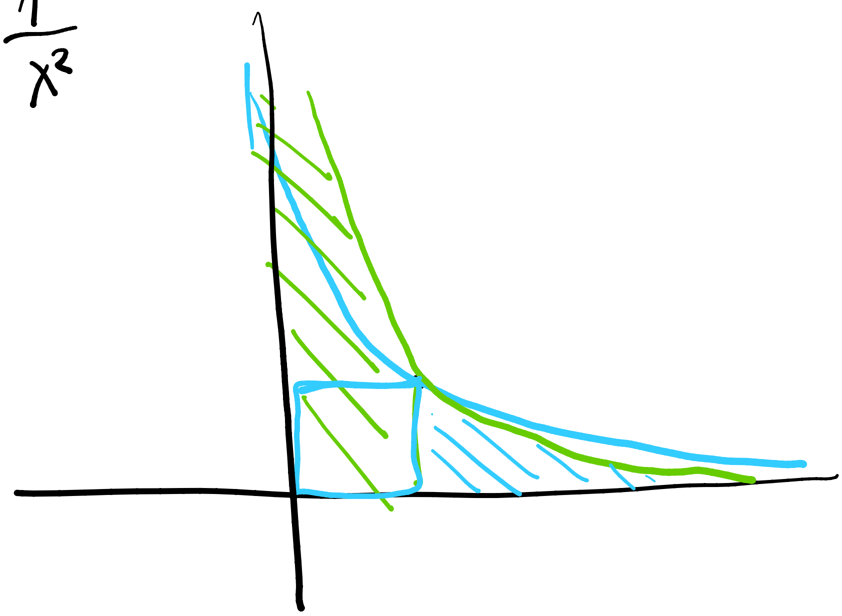
$x \in (0,1]$

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_1^\infty = \lim_{x\to\infty} 2\sqrt{x} - 2 = \underline{\underline{+\infty}}$$

$$\int_1^\infty \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^\infty = \lim_{x\to\infty} -\overset{\overset{0}{/\!/}}{\frac{1}{x}} - (-1) = \underline{\underline{1}}$$

$$\int_1^{\infty} \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^{\infty} = \lim_{x\to\infty} -\frac{1}{x} - (-1) = \underline{\underline{1}}$$

$$x \geq 1 \qquad \frac{1}{\sqrt{x}} \geq \frac{1}{x^2}$$



$$\int_{\cancel{0}}^{\cancel{4\pi}} \frac{1}{1+\sin^2 x}\,dx = \int \frac{1}{\left(\frac{t^2}{1+t^2}+1\right)(1+t^2)}\,dx$$

$$t = \operatorname{tg} x \qquad \sin^2 x = \frac{t^2}{1+t^2} \qquad = \int \frac{1}{2t^2+1}\,dt$$

$$dx = \frac{1}{1+t^2}\,dt \qquad = \frac{1}{\sqrt{2}}\operatorname{arctg}(\sqrt{2}\,t)$$

$$\boxed{t = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi}$$

$$\int_0^{4\pi} \frac{1}{1+\sin^2 x}\,dx = 4\int_0^{\pi} \frac{1}{1+\sin^2 x}\,dx = 4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+\sin^2 x}\,dx$$

II

$$= 8\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin^2 x}\,dx$$