Cvičení 3.11.

úterý 3. listopadu 2020 13:42

$$(1-x)(1-2x)(1-3x)\cdots(1-311x)$$

$$=1+ax+bx^2+\cdots+zx^{311}$$

$$a_1x+a_2x^2+\cdots+a_{311}x^{311}$$

- $(f+g)'=f'+g'$ $\qquad (g\circ f)'=f'\cdot(g'\circ f)$

$(fg)'=f'g+fg'$

- $\left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2}$

$$(\operatorname{arctg} x)'=\frac{1}{1+x^2} \qquad (\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$$

- $(x^n)'=nx^{n-1}$ $\quad n\in\mathbb{N},\ x\in\mathbb{R} \quad (C)'=0 \quad C\in\mathbb{R}$

$$(x^3+2x+1)'=3x^2+2+0$$

$$(x^5+31x^3+x)'=5x^4+31\cdot 3\cdot x^2+1$$

$$(31x^3)=\underbrace{(31)'}_{=0}\cdot(x^3)+31\cdot(x^3)'=31\cdot 3x^2$$

$$\left(\frac{f}{g}\right)'=\frac{f'g-fg'}{}$$

$$\left(\frac{x^2+1}{x-1}\right)' = \frac{2x(x-1)-(x^2+1)\cdot 1}{(x-1)^2} \qquad \left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$$

$$f'(x)=(x^2+1)=2x \qquad g'(x)=(x-1)=1$$

$$\frac{2x^2-2x-x^2-1}{(x-1)^2} = \frac{x^2-2x-1}{(x-1)^2}$$

$x \in \mathbb{R}$ $\quad (\sin x)'=\cos x \quad (\cos x)'=-\sin x \quad (e^x)'=e^x$

$x>0$ $\quad (\log x)' = \frac{1}{x} \qquad f'\cdot g'\circ f = (f\circ g)'$

$$\frac{\sin^2 x}{\cos(x^2)}$$

$(\sin^2 x)' = ?$ (1)

$(\cos(x^2))' = ?$ (2)

$$(\sin^2 x)' = (\sin x \cdot \sin x)' = (\sin x)' \sin x + \sin x \underbrace{(\sin x)'}_{=\cos x}$$

$$= 2 \sin x \cos x \leftarrow \sin(2x)$$

- $(f\circ g)' = f' \cdot g' \circ f$

$f(x)=\sin x \quad f'(x)=\cos x$

$g(x)=x^2 \quad g'(x)=2x$

$$(\sin^2 x)' = \cos x \cdot 2\sin x$$

$$(\cos(x^2))' = 2x\cdot(-\sin(x^2)) = -2x\sin(x^2)$$

$((\cos(x^2))$

$f(x) = x^2 \quad f'(x) = 2x$

$g(x) = \cos x \quad g'(x) = -\sin x$

$$(\sin^2 x)' = 2\sin x\cos x \qquad ((\cos(x^2))' = -2x\sin(x^2)$$

$$\left(\frac{\sin^2 x}{\cos(x^2)}\right)' = \frac{2\sin x\cos x\cdot\cos(x^2) + \sin^2 x\cdot 2x\cdot\sin(x^2)}{\cos^2(x^2)}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\sin\left(e^{(x^2+1)}\right)$$

$f(x) = \sin x \quad g(x) = e^x \quad h(x) = x^2+1$

$$f(g(h(x))) \qquad h'\cdot g'\circ h$$

$$\Downarrow$$

$$(f\circ(g\circ h))' = (g\circ h)'\cdot f'\circ(g\circ h)$$

$$= h'\cdot g'\circ h\cdot f'\circ(g\circ h)$$

$$\left(\sin\left(e^{x^2+1}\right)\right)' = 2x\cdot e^{x^2+1}\cdot\cos\left(e^{x^2+1}\right)$$

$$\left(e^{x^2+1}\right)' = 2x\, e^{x^2+1}$$

$(e^{x^2+1})' = 2x \cdot e^{x^2+1}$

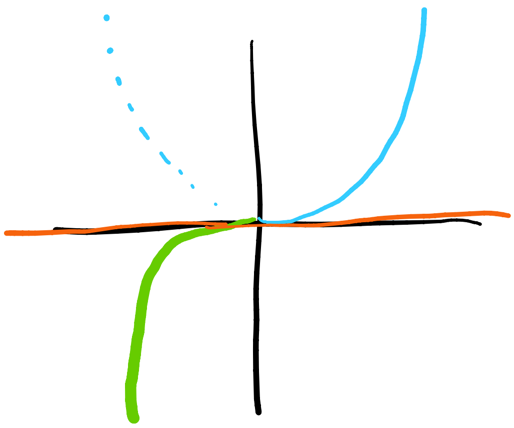
(5)-(10) - mechanické derivování

(1) $f(x) = x \cdot |x|$ $\quad f'(0) = ?$

$(fg)' = f'g + fg'$ $\quad (x \cdot |x|)' = 0$

$(|x|)'_{\pm} = \pm 1$

$$x \cdot |x| = \begin{cases} \boxed{x^2} & x \geq 0 \\ \boxed{-x^2} & x \leq 0 \end{cases}$$



$(x^2)' = 2x$ $\leftarrow$ $\Rightarrow (x^2)'_+ = 2x$

$(-x^2)' = -2x$ $\leftarrow$

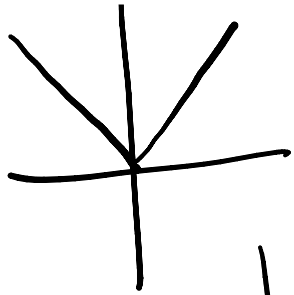
$f(x) = x \cdot |x|$ $\quad f'_-(0) = 0 = f'_+(0) = 0$

$f'(0) = 0$

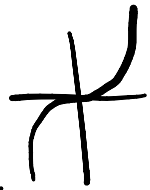
- $f'_-(0) = \lim\limits_{x \to 0-} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0-} \dfrac{x \cdot |x|}{x}$

$= \lim\limits_{x \to 0-} |x| = 0$

$$= \lim_{x \to 0-} |x| = 0$$



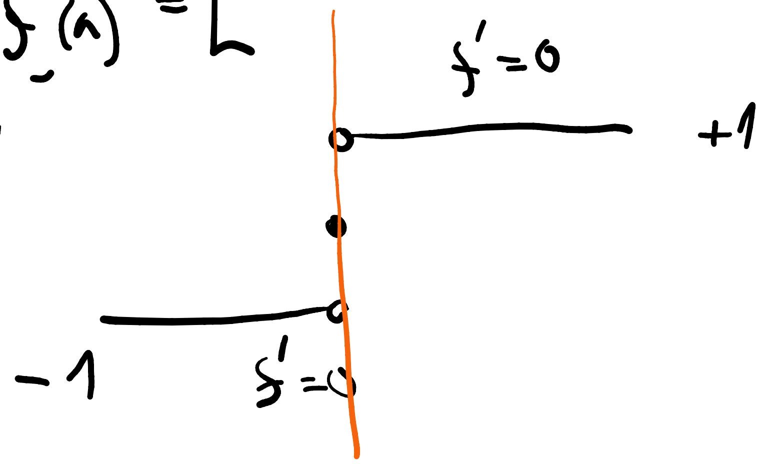
$$(|x|)' = \operatorname{sgn} x \qquad x \neq 0$$



Pokud f je spojitá zprava/zleva v a a existuj $\lim_{x \to a+} f'(x)$ / $\lim_{x \to a-} f'(x) = L$

Potom $f'_+(a)$ / $f'_-(a) = L$

$$(\operatorname{sgn} x)'$$



$x \cdot |x|$ spojitá v 0

$x \neq 0$ $\quad (x \cdot |x|)' = (x)' \cdot |x| + x \cdot (|x|)'$

$$= 1 \cdot |x| + \underbrace{x \cdot \operatorname{sgn} x}_{= |x|} = 2|x|$$

$$f'_-(0) = \lim_{x \to 0-} f'(x) = \lim_{x \to 0-} 2|x| = 0$$

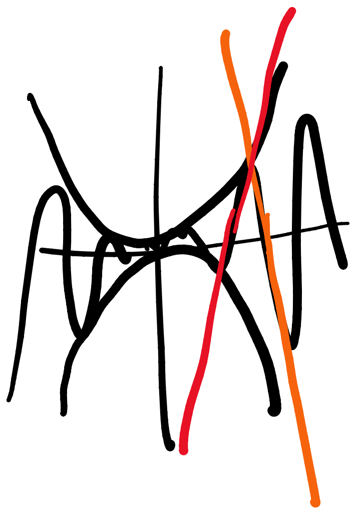
$$f'(0) = 0 \Rightarrow f'(0) = 0$$

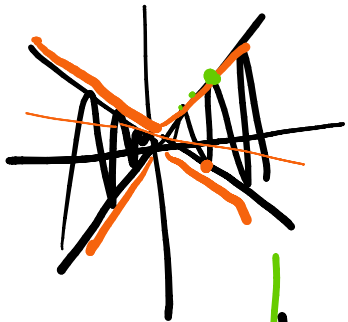
$$f'_\pm(0)=0 \Rightarrow f'(0)=0$$

(3)
$$f(x)=\begin{cases} |x|^\alpha \sin\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$$

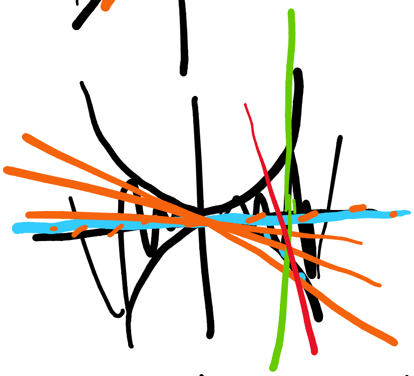
$\alpha\in\mathbb{R}$ $\quad \alpha=1$

$|x|\sin\frac{1}{x}$





$|x|^2\sin\frac{1}{x}$

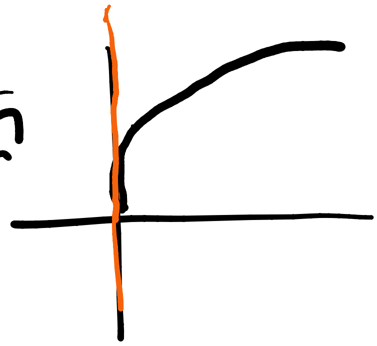


(11) $f(x)=\sqrt{1-e^{-x^2}}$ $\qquad$ • $\left(x^{\frac{1}{2}}\right)'=\frac{1}{2}\cdot x^{-\frac{1}{2}}$

$x\neq 0$

$e^{-x^2}<1$

$$f'(x)=\left(1-e^{-x^2}\right)'\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{1-e^{-x^2}}}$$



$$=\frac{-(-2x)\,e^{-x^2}}{2\sqrt{1-e^{-x^2}}}$$

spojitá

$$f'(x)=\boxed{\frac{x e^{-x^2}}{\sqrt{1-e^{-x^2}}}} \qquad x\neq 0$$

$$\sqrt{1-e^{-x^2}}$$

$$f_+'(0)=\lim_{x\to 0+}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0+}\frac{\sqrt{1-e^{-x^2}}-0}{x-0}$$

$$=\lim_{x\to 0+}\frac{\sqrt{1-e^{-x^2}}}{x}=\lim_{x\to 0+}\sqrt{\frac{1-e^{-x^2}}{x^2}}$$

$$\boxed{\lim_{x\to 0}\frac{e^{x}-1}{x}}$$

$$x>0 \Rightarrow \sqrt{x^2}=x$$

$\sqrt{}$ spojita v 1

$$-x^2=0 \Leftrightarrow x=0$$

$$=\lim_{x\to 0+}\sqrt{\frac{e^{-x^2}-1}{-x^2}}=\sqrt{\lim_{x\to 0}\frac{e^{-x^2}-1}{-x^2}}=\sqrt{1}=1$$

$$\boxed{f_+'(0)=1}\neq\boxed{f_-'(0)=-1}$$

$f'(0)$ neexistuje

$$f_-'(0)=\lim_{x\to 0-}\frac{\sqrt{1-e^{-x^2}}}{x}=\lim_{x\to 0-}-\sqrt{\frac{1-e^{-x^2}}{x^2}}$$

$$x\to 0^- \Rightarrow x<0 \Rightarrow \sqrt{x^2}=-x$$

$$=-\lim_{x\to 0-}\sqrt{\frac{e^{-x^2}-1}{-x^2}}=-1$$

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(5)-(10) (12),(13) $f_\pm'$

$(\sqrt{})^{(10)}$

(14) $\left(\sqrt{x}\right)^{(10)}$

(15) $f(x) = \underbrace{(x^4 + 3x)\sin x}$