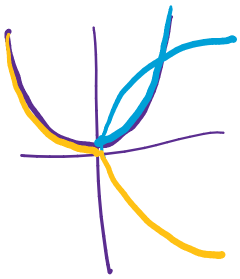
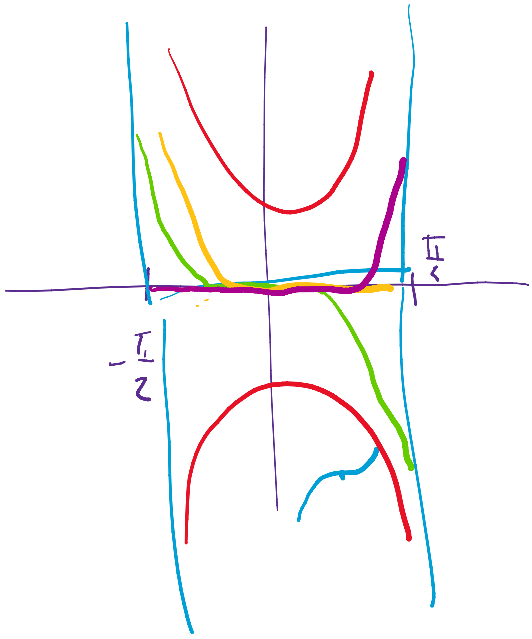
středa 31. března 2021 12:13

$J$

$\vee$

$(0,\infty)$






$$y(x) = \begin{cases} y_1(x) & \left(-\frac{\pi}{2}, -\arccos e^{c}\right) \quad X \\ 0 & \left[-\arccos e^{c}, \arccos e^{D}\right] \\ y_2(x) & \left(\arccos e^{D}, \frac{\pi}{2}\right) \quad X \end{cases}$$

- $\tilde{y}(x) = y(x + k\pi)$

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(L) $a_n y^{(n)} + \dots + a_0 y = f \qquad a_0, \dots, a_n \in \mathbb{R}$

$y = y_p + y_h$

$y_h$ → řešení (Lh) $f = 0$

$$a_n y^{(n)} + \dots + a_0 y = 0$$ – homogenní rovnice

$\downarrow$

$$p(\lambda) = a_n \lambda^n + \dots + a_0 = 0$$

$a$ reálný kořen násobnosti $k$, potom do FS přidáme

$$\underbrace{\{e^{ax}, xe^{ax}, \dots, x^{k-1}e^{ax}\}}_{k \text{ funkcí}} \to FS$$

$$y'' - y = 0 \to \lambda^2 - 1 = (\lambda-1)(\lambda+1) \to \lambda_{1,2} = \pm 1$$

$$\to \{e^x, e^{-x}\}$$

$$(\lambda^2-1)^2 = (\lambda-1)^2(\lambda+1)^2 \to \lambda_{1,2} = \pm 1, \quad \lambda_{3,4} = \pm 1$$

$$\to \{e^x, xe^x, e^{-x}, xe^{-x}\}$$

komplexní kořeny $\alpha \pm \beta i$ násobnosti $k$

$k$ $\{e^{\alpha x}\cos\beta x, xe^{\alpha x}\cos\beta x, \dots, x^{k-1}e^{\alpha x}\cos\beta x,$

$k$ $e^{\alpha x}\sin\beta x, xe^{\alpha x}\sin\beta x, \dots, x^{k-1}e^{\alpha x}\sin\beta x\} \to FS$

$2k$ funkcí

$$y'' + y = 0 \qquad \lambda^2 + 1 = 0 \qquad \lambda_{1,2} = \pm i$$

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$FS = \{\sin x, \cos x\} \qquad \alpha = 0,\ \beta = 1$

$y = A\sin x + B\cos x \qquad A, B \in \mathbb{R}$

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$y'' - 3y' + 2y = 0 \rightarrow FS = \{e^x, e^{2x}\}$

$\lambda^2 - 3\lambda + 2 = (\lambda - 1)(\lambda - 2)$   1   2

$y'' + 6y' + 9y = 0 \rightarrow FS = \{e^{-3x}, xe^{-3x}\}$

$\lambda^2 + 6\lambda + 9 = (\lambda + 3)^2 \rightarrow$

$-3, -3$

$y'' + 3y = 0 \rightarrow FS = \{\sin\sqrt{3}x, \cos\sqrt{3}x\}$

$\lambda^2 + 3 = (\lambda + \sqrt{3}i)(\lambda - \sqrt{3}i)$

$\underset{\alpha}{0} \pm \underset{\beta}{\sqrt{3}}i \rightarrow e^{\alpha x} \begin{cases} \sin\beta x \\ \cos\beta x \end{cases}$

$y'''' + 8y'' + 16y = 0 \rightarrow FS = \{\cos 2x, \sin 2x, x\cos 2x, x\sin 2x\}$

$\lambda^4 + 8\lambda^2 + 16 = (\lambda^2 + 4)^2 = (\lambda - 2i)^2(\lambda + 2i)^2$

$2i \qquad -2i$

$t = \lambda^2 \qquad t^2 + 8t + 16 = (t+4)^2 \qquad \lambda^2 + 4 = (\lambda - 2i)(\lambda + 2i)$

$2 \cdot 4 \quad 4^2$

$\alpha = 0 \quad \beta = 2$

2.4 4°

$\alpha = 0 \quad \beta = 2$

$$y^{(5)} - 6y^{(4)} + 12y''' - 8y'' = 0 \Rightarrow FS = \{1, x, e^{2x}, xe^{2x}, x^2e^{2x}\}$$

$$\lambda^5 - 6\lambda^4 + 12\lambda^3 - 8\lambda^2 = \lambda^2(\lambda^3 - 6\lambda^2 + 12\lambda - 8)$$

$\lambda = 2$ násobnost 3

$\lambda = 0$ násobnost 2 $\quad e^{0x}, xe^{0x}$

$$y''' - y'' - 3y' + 27y = 0 \Rightarrow FS = \{e^{-3x}, e^{2x}\sin\sqrt{5}x, e^{2x}\cos\sqrt{5}x\}$$

$$\lambda^3 - \lambda^2 - 3\lambda + 27 = (\lambda+3)(\lambda^2 - 4\lambda + 9) = $$

a $\quad \alpha, \beta$

$\lambda = -3 \qquad \lambda^3 + 3\lambda^2 - 3\lambda^2 - \lambda^2 - 3\lambda + 27 \quad 9\lambda + 27$

$$\lambda^2(\lambda+3) - 4\lambda^2 - 12\lambda + 12\lambda - 3\lambda + 27$$

$$\lambda^2(\lambda+3) - 4\lambda(\lambda+3) + 9(\lambda+3)$$

$$\lambda^2 - 4\lambda + 9 \qquad \lambda_{1,2} = \frac{4 \pm \sqrt{16-36}}{2} = 2 \pm \sqrt{5} \qquad \sqrt{20}$$

$\alpha = 2 \quad \beta = \sqrt{5}$

$$y(x) = Ae^{3x} + Be^{2x}\sin\sqrt{5}x + Ce^{2x}\cos\sqrt{5}x$$

$$S(x) = \overset{0}{P(x)}\, e^{\alpha x}\sin\beta x + Q(x)\, e^{\alpha x}\cos\beta x$$

$=0 \qquad \cos 0 = 1$

speciální pravá strana

$$y_p(x) = x^k\left(R(x)e^{\alpha x}\sin\beta x + S(x)e^{\alpha x}\cos\beta x\right)$$

[illegible] $\alpha + \beta i$ v $P(\lambda)$

$y_p(x) = x^k \dots$

k je násobnost kořenu $\alpha+\beta i$ v $p(\lambda)$

homogenní rovnice

$R(x), S(x)$ jsou polynomy stupně nejvýše $\max(\deg P, \deg Q)$

$e^{-x}$

$y''-3y'+2y = xe^{-x}$ $\alpha=-1, \beta=0$

$\lambda_{1,2}$ $Q(x)=x$ $\deg Q = 1$

$$y_p(x) = (Ax+B)e^{\alpha x}\overset{0}{\sin\beta x} + (Cx+D)e^{\alpha x}\overset{1}{\cos\beta x}$$
$$=(Cx+D)e^{-x}$$

- $y_p(x) = (Cx+D)e^{-x}$
- $y_p'(x) = Ce^{-x} - (Cx+D)e^{-x} = e^{-x}(C-D-Cx)$
- $y_p''(x) = -Ce^{-x} - Ce^{-x} + (Cx+D)e^{-x}$
$$= e^{-x}(D-2C+Cx)$$
- $x\cancel{e^{-x}} = \cancel{e^{-x}}(D-2C+Cx) - 3\cancel{e^{-x}}(C-D-Cx)$
$$+2\cancel{e^{-x}}(D+Cx)$$

$0+1\cdot x = (D-2C-3C+3D+2D)+(C+3C+2C)x$

$0 = 6D-5C \rightarrow D=\frac{5}{6}C=\frac{5}{36}$

$0 = 6D - 5C \Rightarrow D = \frac{5}{6}C = \frac{5}{36}$

$1 = 6C \Rightarrow C = \frac{1}{6}$

$$y''' + 4y'' + 4y' = \underbrace{(x+3)}_{Q(x)} e^{-2x} \quad \cos\beta x$$

$\boxed{\alpha = -2}$ $\boxed{\beta = 0}$

$$\boxed{y_p(x) = x^2\left[(Ax+B)e^{-2x}\right]}$$

$$\lambda^3 + 4\lambda^2 + 4\lambda = \lambda(\lambda^2 + 4\lambda + 4) = \lambda(\lambda+2)^2$$

nas. 2 $\boxed{\alpha = -2 \quad (\beta = 0)}$

$$y'' + 9 = x^2 \sin 3x + 0 \cos$$

$0$

$\lambda^2 + 9 = 0$ $\qquad P(x)\cancel{e^{\alpha x}}\sin\beta x + Q(x)e^{\alpha x}\cos\beta x$

$\lambda_{1,2} = \pm 3i$ $\quad$ nas. 1 $\quad \alpha = 0 \quad \beta = 3$

$\alpha = 0, \beta = 3$ $\quad P(x) = x^2 \quad \deg P = 2$

$$y_p(x) = x\left[(Ax^2 + Bx + C)\sin 3x + (Dx^2 + Ex + F)\cos 3x\right]$$

$$y'' + 9 = \sin 3x \, e^{5x}$$

$\alpha = 5$

$\beta=3$ $\quad \alpha=5$

$$\boxed{y^{(4)}+4y''}=\boxed{x^2-1}+\boxed{3\cos x} \qquad y_{p1}+y_{p2}$$

$Q(x)$, $\alpha=0,\beta=0$ $\qquad$ $\alpha=0 \quad \beta=1$

Variace konstant

$a_2=1$

- $y''-2y'+y=\dfrac{e^x}{x}$ $\quad \mathscr{I}$ $\quad (-\infty,0),(0,\infty)$

- $y_p(x)=C(x)\,y_1(x)+D(x)\,y_2(x)$

$\{y_1,y_2\}\to$ FS homogenní rovnice

$\{e^x, xe^x\}$ $\qquad (xe^x)'=e^x+xe^x=(1+x)e^x$

$$\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix}\begin{pmatrix} C' \\ D' \end{pmatrix}=\begin{pmatrix} 0 \\ \frac{f}{a_2} \end{pmatrix}$$

$$\begin{pmatrix} e^x & xe^x \\ e^x & (1+x)e^x \end{pmatrix}\begin{pmatrix} C' \\ D' \end{pmatrix}=\begin{pmatrix} 0 \\ \frac{e^x}{x} \end{pmatrix}$$

$$\begin{cases} e^x C' + x e^x D' = 0 \\ e^x C' + (1+x) e^x D' = \dfrac{e^x}{x} \end{cases}$$

$$\begin{cases} C' + x D' = 0 \\ C' + (1+x) D' = \dfrac{1}{x} \end{cases}$$

$$D' = \frac{1}{x} \qquad C' = -1$$

$$\Rightarrow \boxed{D = \log|x| \qquad C = -x}$$

$$\boxed{y_p(x) = -x e^x + \log|x| \cdot x e^x} \qquad x \in (-\infty, 0), (0, \infty)$$