27.10.

úterý 27. října 2020 13:42

- $\lim_{x\to 0} \frac{\sin x}{x} = 1$
- $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$
- $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x\to 0} \frac{\log(x+1)}{x} = 1$
- $\lim_{x\to 1} \frac{\log x}{x-1}$

Věta o limitě složené funkce (S) g spojitá v B

$\lim_{x\to A} g\circ f(x) = C$ (P) $\exists \delta > 0\ \forall x \in P(A,\delta): f(x) \neq B$

$n, m \in \mathbb{N}$.

- $\lim_{x\to 0} \frac{\sin nx}{\sin mx} \cdot \frac{nx}{nx} \cdot \frac{mx}{mx} = \lim_{x\to 0} \frac{\sin nx}{nx} \cdot \frac{mx}{\sin mx} \cdot \frac{n}{m} = 1 \cdot 1 \cdot \frac{n}{m}$

- $g(x) = \frac{\sin x}{x}$
- $\lim_{x\to 0} \frac{\sin nx}{nx} = 1$

$A = 0, B = 0, C = 1$

$f(x) = nx \cdot$
$= mx$

$nx = 0 \iff x = 0$

$nx \neq 0$ pro $x \in P(0,1)$

- $\lim_{x\to 0} \frac{\sin mx}{mx} = 1$

$\lim_{x\to B} g(x) = 1$

$\lim_{x\to A=0} nx = B = 0$

$= \frac{n}{m} \lim_{x\to 0} \frac{\sin nx}{nx} \cdot \lim_{x\to 0} \frac{1}{\frac{\sin mx}{mx}} = \frac{n}{m} \cdot 1 \cdot \frac{1}{1} = \frac{n}{m}$

5) $\lim_{x\to 0+} (1+x)^{\frac{1}{x}} = \lim_{x\to 0+} e^{\frac{1}{x}\log(x+1)} = e \ (=C)$

- $X = e^{\log X} \longrightarrow \boxed{g(x)^{f(x)}} = e^{\log g(x)^{f(x)}} = e^{f(x)\log g(x)}$

$g(x) = e^x \quad f(x) = \dfrac{\log(x+1)}{x}$

$\lim_{x\to 0+} \dfrac{\log(x+1)}{x} = 1$

e^x spojitá na $\mathbb{R}$

$\lim_{x\to 0} f(x) = 1 = B$ $\quad \lim_{x\to 1} g(x) = g(1) = e$

$A = 0$

$B = 1$

$C = e$

$\lim_{x\to 0} g\circ f(x) = e$

VOAL

(1)-(3) úpravy

(6)-(10) VOLSF

- (7),(8),(10) $f(x)^{g(x)}$ (6), (9)

- $\dfrac{e^{\alpha x} - e^{\beta x} + 1 - 1}{\sin\alpha x - \sin\beta x} = \dfrac{e^{\alpha x} - 1}{\sin\alpha x - \sin\beta x} - \dfrac{e^{\beta x} - 1}{\sin\alpha x - \sin\beta x}$

$e^{\alpha x}\left[\dfrac{e^{\beta x}}{e^{\beta x}}\right] - \left[e^{\beta x}\right]_1$

$\dfrac{e^{\alpha x} - 1}{\alpha x}\,\alpha$

$e^{\beta x}\,\dfrac{e^{(\alpha-\beta)x} - 1}{\sin\alpha x - \sin\beta x}$

$\alpha\,\dfrac{\sin\alpha x}{\alpha x} - \dfrac{\sin\beta x}{\beta x}\,\beta$

$\to 1$

$$\lim_{x \to \frac{\pi}{2}} \operatorname{tg} x \log \sin x = \lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\cos x} \cdot \frac{\log \sin x}{\sin x - 1} \cdot (\sin x - 1)$$

$\frac{\log \sin x}{\sin x - 1} \to 1$

$\sin^2 x + \cos^2 x = 1$

$$\frac{\left| \frac{\sin x}{\cos x} - \sin x \right|}{x^3} = \frac{\sin x}{x} \left(\frac{1}{\cos x} - 1 \right) \frac{1}{x^2}$$

$$= \frac{\sin x}{x} \cdot \frac{1 - \cos x}{\cos x \cdot x^2}$$

$\frac{\sin x}{x} \to 1$, $\frac{1 - \cos x}{x^2} \to \frac{1}{2}$, $\cos x \to 1$

$$\sin x^{\operatorname{tg} x} = e^{\operatorname{tg} x \log \sin x}$$

$$= e^{\operatorname{tg} x \cdot \frac{\log \sin x}{\sin x - 1} \cdot (\sin x - 1)}$$

$\frac{\log \sin x}{\sin x - 1} \to 1$

$\sin^2 x + \cos^2 x = 1$

$\frac{\log x}{x - 1}$