Cvičení 26.3.

pátek 26. března 2021 11:27

$$\bullet\ xy' + y = y^2 \rightarrow y' = \frac{y^2 - y}{x} \qquad y' = f(x,y)$$

$y_1 = 0$, $y_2 = 1$ $\quad x \in (-\infty, 0), (0, \infty)$

$\bullet\ y_3 = \frac{1}{1+e^c x}$, $x \in (-\infty, -e^{-c})$

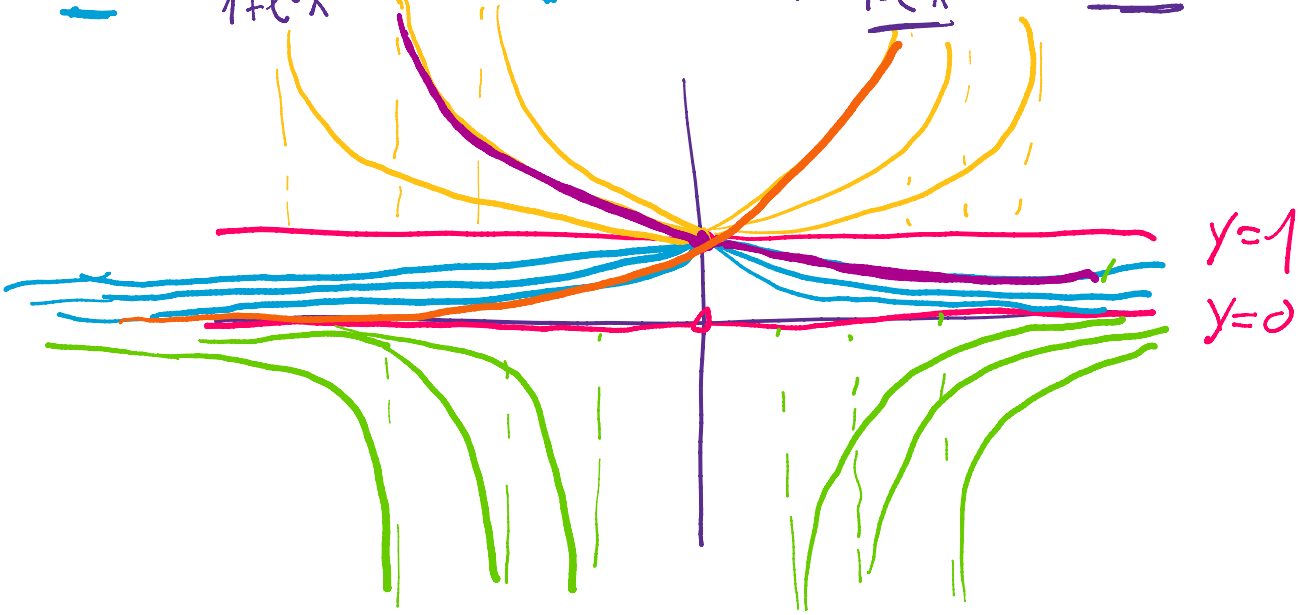
$\bullet\ y_4 = \frac{1}{1-e^c x}$, $x \in (-\infty, 0)$

$\bullet\ y_5 = \frac{1}{1+e^c x}$, $x \in (-e^{-c}, 0)$

$\bullet\ y_6 = \frac{1}{1-e^c x}$, $x \in (e^{-c}, \infty)$

$\bullet\ y_7 = \frac{1}{1+e^c x}$, $x \in (0, \infty)$

$\bullet\ y_8 = \frac{1}{1-e^c x}$, $x \in (0, e^{-c})$



$$\left(\frac{1}{1+e^c x}\right)' = \frac{-e^c}{(1+e^c x)^2} \xrightarrow{x \to 0\pm} -e^c < 0$$

$$\left(\frac{1}{1-e^c x}\right)' = \frac{e^c}{(1-e^c x)^2} \xrightarrow{x \to 0\pm} e^c > 0$$

$\tilde{y}_1(x) = 0 \qquad x \in (-\infty, \infty)$

$\tilde{y}_2(x) = 1 \qquad x \in (-\infty, \infty)$

$y_2(x) = 1 \qquad x \in (-\infty, \infty)$

$$\hat{y}_5(x) = \frac{1}{1-e^c x} \qquad x \in (-\infty, e^{-c})$$

$$\hat{y}_4(x) = \frac{1}{1+e^c x} \qquad x \in (-e^{-c}, \infty)$$

y_3, y_6

$$xy' + y = y^2$$

$$\underset{0}{0 \cdot y'(0)} + \underset{1}{y(0)} = \underset{1}{y(0)^2} \checkmark$$

$$x \cdot 1 = 2x - x \checkmark \qquad 0 = 0 - y(0)$$

(3) $\quad xy' = 2x - y \qquad y' = F(x,y) \Rightarrow F(\lambda x, \lambda y) = F(x,y)$

$\lambda \neq 0$

$$F\left(\tfrac{1}{x}x, \tfrac{1}{x}y\right) = F\left(1, \tfrac{y}{x}\right)$$

$$y' = 2 - \boxed{\frac{y}{x}}$$

$$z = \frac{y}{x} \qquad y = xz \qquad y' = xz' + z$$

$$xz' + z = 2 - z \rightarrow z' = 2\frac{1-z}{x} \qquad f(x) = \frac{2}{x} \quad g(z) = 1-z$$

(1) $I_1 = (-\infty, 0), I_2 = (0, \infty)$

(2) $J_1 = (-\infty, 1), J_2 = (1, \infty)$

(3) $z_1 = 1 \qquad (0,\infty), (-\infty, 0)$

(3) $z_0 = 1 \quad (0,\infty),(-\infty,0)$

(4) $F(x) = \log(x^2) \qquad G(z) \overset{c}{=} \int \frac{1}{1-z} \overset{c}{=} -\log|1-z|$

$J_1 \to G(z) = -\log(1-z) \qquad -\infty \to -\infty \qquad 1 \to \infty$

$J_2 \to G(z) = -\log(z-1)$

(5) $G^{-1}(F(x)+C)$

$t = -\log(z-1) \qquad e^{t} = z-1$

$z = 1+e^{t}$

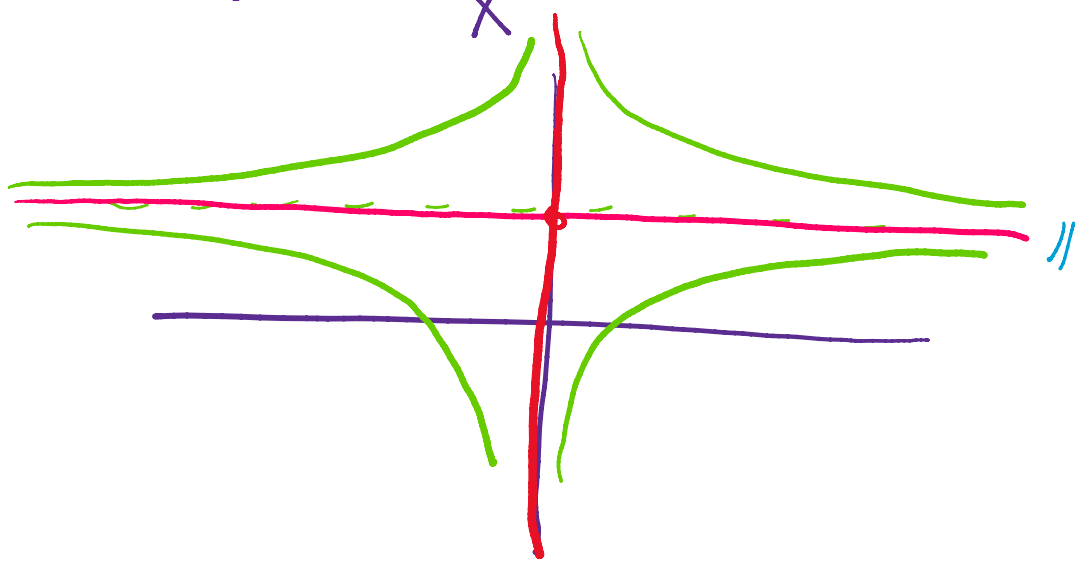
$G(J_1) = \mathbb{R} \qquad G(J_2) = \mathbb{R}$

$t = -\log(1-z) \to e^{-t} = 1-z \to z = 1-e^{-t}$

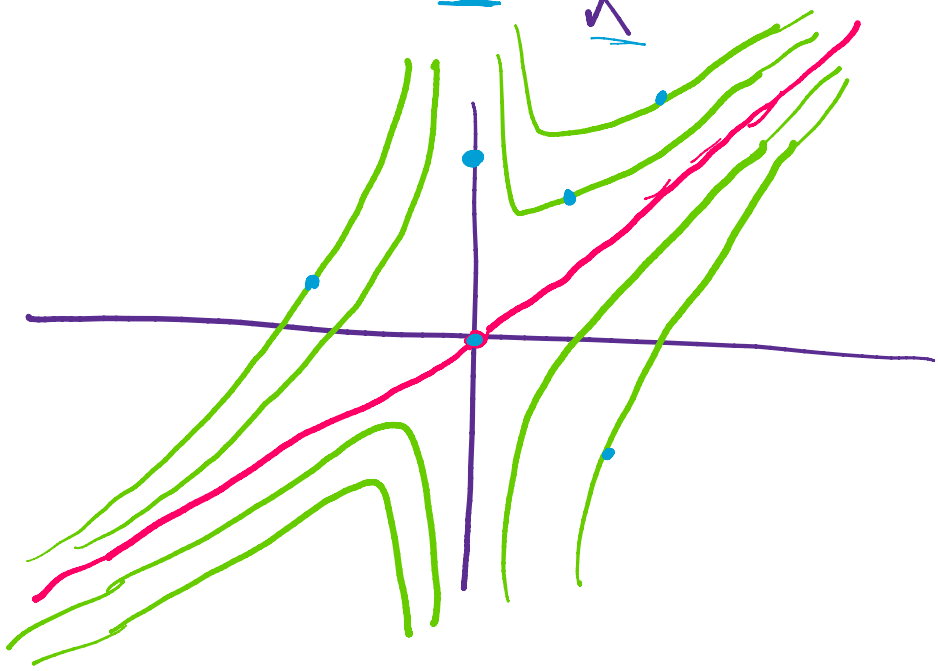
$z_1(x) = 1 - e^{-(\log(x^2)+C)} = 1 - \frac{e^{-C}}{x^2} \qquad x \in (-\infty,0),(0,\infty)$

$z_2(x) = 1 + \frac{e^{-C}}{x^2} \qquad x \in (-\infty,0),(0,\infty)$

$\tilde{z}(x) = 1 + \frac{D}{x^2} \qquad x \in (-\infty,0),(0,\infty)$



$$y(x) = x\tilde{z}(x) = \underline{x} + \frac{D}{\underline{x}} \qquad x \in (-\infty, 0),\ (0, \infty)$$



$$\tilde{y}(x) = x$$

$$0\ y'(0) = 0$$

(4) $x^2 y' = y(x-y) \quad \to \quad y' = \frac{y}{x}\left(1 - \frac{y}{x}\right)$

$$z = \frac{y}{x} \qquad y = z \cdot x \qquad y' = z'x + z$$

- $z'x + \underline{z} = z(1-z) = \underline{z} - z^2$

$$z' = -\frac{z^2}{x} \qquad f(x) = \frac{1}{x},\ g(z) = -z^2$$

(1) $I_1 = (-\infty, 0),\ I_2 = (0, \infty)$

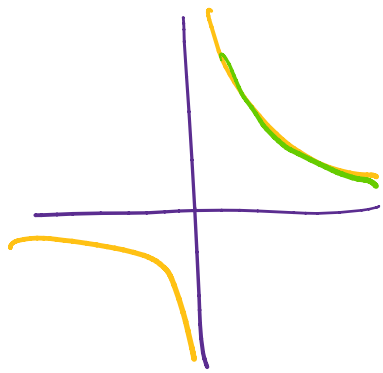
(2) $J_1 = (-\infty, 0),\ J_2 = (0, \infty)$

(2) $J_1 = (-\infty, 0)$, $J_2 = (0, \infty)$

(3) • $z_0 = 0$ $\quad \underline{I_1}, \underline{I_2}$

(4) $F(x) = \log|x| \begin{cases} \log x & x \in I_2 \\ \underline{\log(-x) \quad x \in I_1} \end{cases}$

$G(z) = \frac{1}{z} \qquad J_1, J_2$



(5) $G(J_1) = (-\infty, 0)$

I_1, J_1

$F(x) + C \in (-\infty, 0)$ $\quad (0, \infty)$

$\log(-x) + C \in (-\infty, 0)$ $\quad 0, +\infty$ $\leftrightarrow \log(-x) \in (-\infty, -C)$

$\leftrightarrow -x \in (0, e^{-C}) \leftrightarrow x \in (-e^{C}, 0)$

$G^{-1}(t) = \frac{1}{t}$

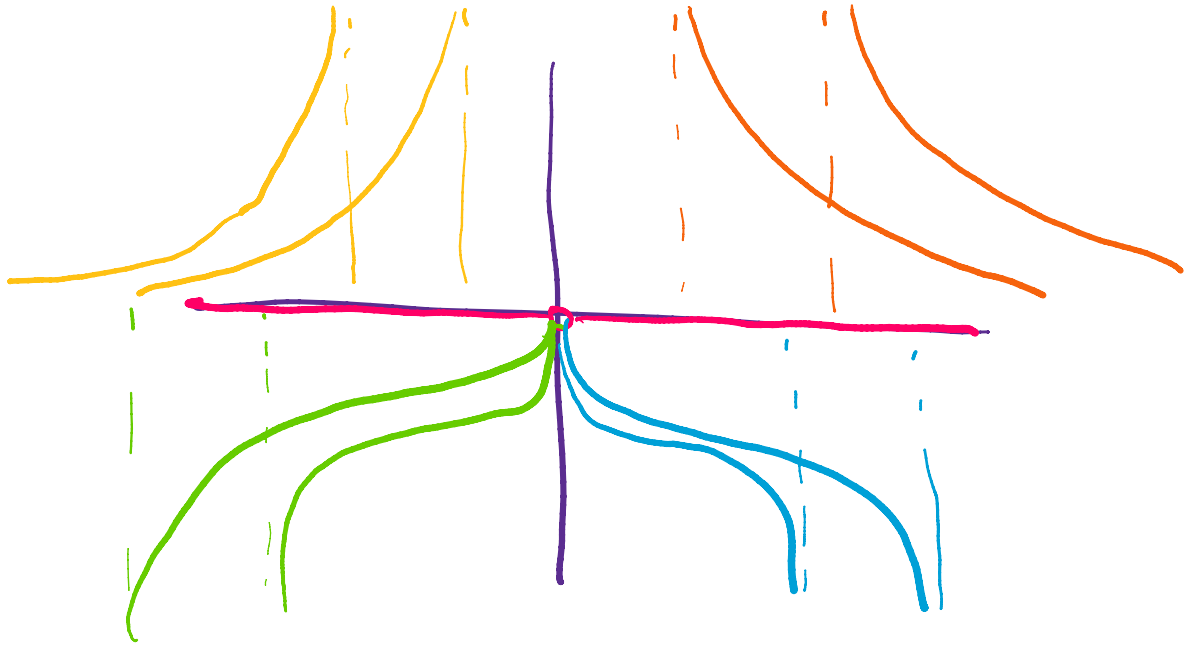
I_1, J_2

• $z_1(x) = \dfrac{1}{\log(-x) + C} \qquad x \in (-e^{C}, 0)$

• $z_2(x) = \dfrac{1}{\log(-x) + C} \qquad x \in (-\infty, -e^{-C})$

• $z_3(x) = \dfrac{1}{\log x + C} \qquad x \in (0, e^{-C})$

$$\frac{1}{\log x + C} \qquad x \in (e^{-C}, \infty)$$

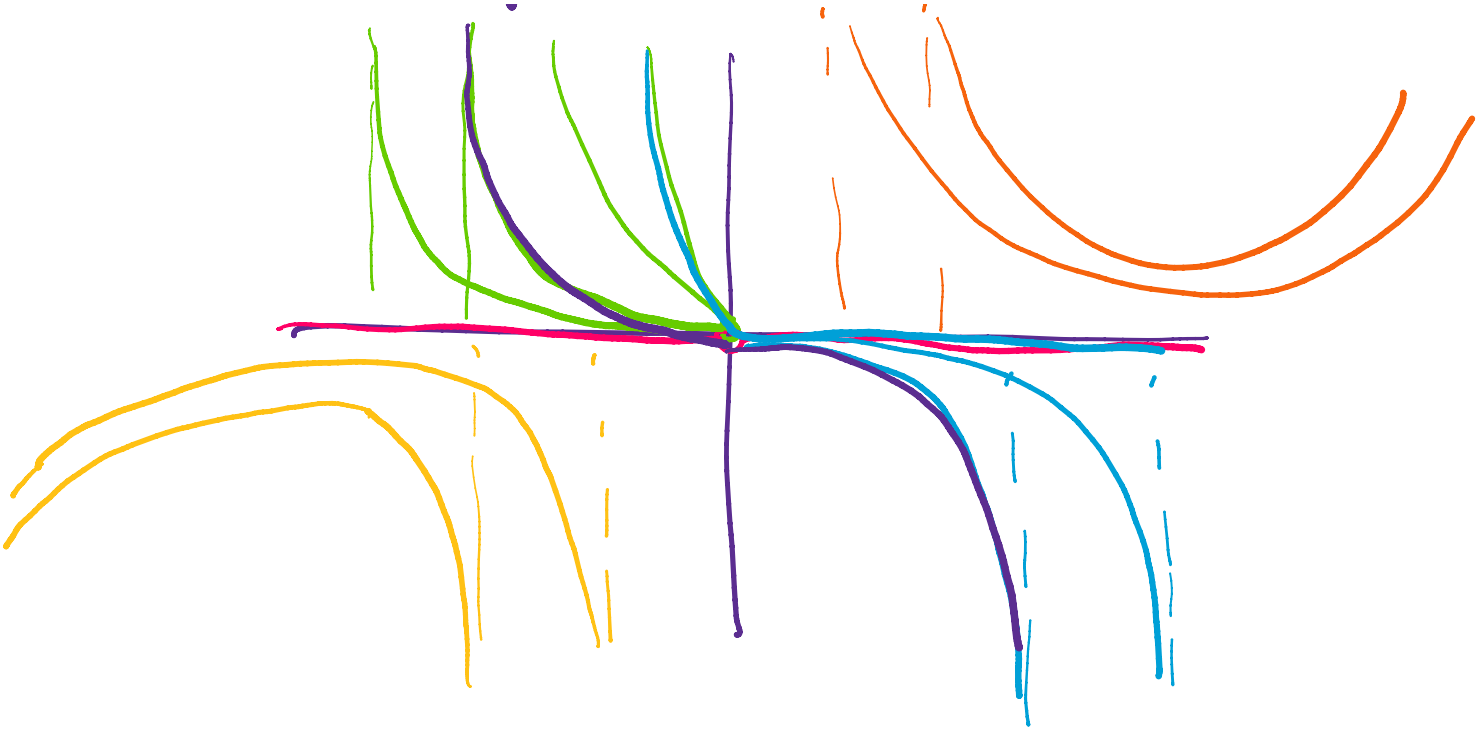


$$\left(\frac{1}{\log x + C}\right)' = \frac{-\frac{1}{x}}{(\log x + C)^2} \xrightarrow{x \to 0^+} -\infty$$

$y_0 = 0, \quad I_1, I_2$

$$y_1(x) = \frac{x}{\log(-x) + C} \qquad x \in (-e^{C}, 0)$$

$$y_2(x) = \frac{x}{\log(-x) + C} \qquad x \in (-\infty, -e^{C})$$



$$\left(\frac{x}{\log x + C}\right)' = \frac{\log x + C - 1}{(\log x + C)^2} \xrightarrow{x \to 0^+} 0$$

- $y' = 1 - y$ $\qquad$ $y' + a(x)y = b(x)$

$y' + y = 1$ $\qquad$ $a(x) = 1,\ b(x) = 1$

$$\boxed{A(x) \overset{c}{=} \int a(x) = \int 1 = x} \rightarrow \boxed{e^{A(x)} = e^{x}}$$

$$\boxed{B(x) \overset{c}{=} \int b(x) e^{A(x)} = \int e^{x} = e^{x}}$$

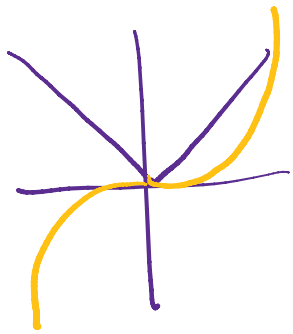
$$\boxed{y(x)} = B(x) e^{-A(x)} + C e^{-A(x)}$$

$$= \boxed{1 + C e^{-x}} \qquad x \in (-\infty, \infty)$$

$$= \boxed{1 + Ce^{-x}} \qquad x \in (-\infty, \infty)$$

(2) $\quad xy' = 2x - y \rightarrow y' + \frac{y}{x} = 2 \qquad (-\infty,0),(0,\infty)$

$$a(x) = \frac{1}{x} \qquad b(x) = 2$$



$$A(x) = \log|x| = e^{A(x)} = |x|$$

$$B(x) = \int e^{A(x)} \cdot b(x) = 2\int |x| = x|x|$$

$$y(x) = B(x)e^{-A(x)} + Ce^{-A(x)} = x + C\underbrace{\frac{1}{|x|}}_{\operatorname{sgn} x \cdot x^{-1}} \qquad x \in (-\infty,0),(0,\infty) \quad C \in \mathbb{R}$$

$$= x + \frac{C}{x}$$

$$\cdot \frac{C}{|x|} \rightarrow \frac{C}{\operatorname{sgn} x} \cdot \frac{1}{x} \qquad D = -C$$

$$\rightarrow -\frac{C}{x} \rightarrow \frac{D}{x}$$

(7) $\quad y' = y + y^2 \rightarrow y' + p(x)y = q(x)y^\alpha \qquad \alpha \neq 1$

$$y' - y = y^2 \qquad p(x) = -1, \; q(x) = 1$$

$$z \qquad \alpha = 2 \qquad 1 - \alpha = -1$$

$$y' \quad 1 \quad y'$$

$$\boxed{-\frac{y'}{y^2}} + \boxed{\frac{1}{y}} = -1 \qquad \left(\frac{1}{y}\right)' = -\frac{y'}{y^2}$$

z'

$$z = \frac{1}{y} \qquad (z = y^{1-\alpha})$$

$$z' + z = -1 \rightarrow \quad a(x) = 1 \quad b(x) = -1$$

$$A(x) = x \quad e^{A(x)} = e^x$$

$$B(x) \overset{c}{=} \int -e^x = -e^x$$

$$z(x) = B(x)\, e^{-A(x)} + C e^{-A(x)} = -e^x \cdot e^{-x} + C e^{-x}$$

$$= -1 + C e^{-x}$$

$$x \in \mathbb{R}$$

$$y = \frac{1}{Ce^{-x} - 1}$$

$$Ce^{-x} - 1 = 0$$

$$e^{-x} = \frac{1}{C}$$

$$-x = \log \frac{1}{C}$$

$$x = \log C$$

$$x \in (-\infty, \log C), \quad (\log C, \infty)$$

(a)

$$y' + 4\frac{y}{x} = 2x\sqrt{y} \qquad y > 0$$

$$\alpha = \frac{1}{2} \qquad 1 - \alpha = \frac{1}{2}$$

$$\boxed{y'} \quad (\sqrt{y})' = z$$

$$\boxed{\frac{1}{2}\frac{y'}{\sqrt{y}}} + 2\frac{\overbrace{(\sqrt{y})}^{=z}}{x} = x \rightarrow \underline{z' + \frac{2z}{x} = x}$$

$$(\sqrt{y})' = \frac{1}{2}\cdot\frac{1}{\sqrt{y}}y'$$

$$a(x) = \frac{2}{x} \qquad A = \log(x^2) \qquad \boxed{e^{A(x)} = x^2}$$

$$b = x \qquad B(x) \overset{c}{=} \int x \cdot x^2 = \frac{x^4}{4} \rightarrow \boxed{B(x) = \frac{x^4}{4}}$$

$$z(x) = \frac{x^2}{4} + \frac{C}{x^2} = \underline{\frac{x^4 + 4C}{4x^2}}$$

$$y = z^2 \qquad y = \left(\frac{x^4 + 4C}{\underbrace{4x^2}_{>0}}\right)^2$$

$$\underset{0}{x^4} + 4C > 0 \longrightarrow$$