neděle 22. listopadu 2020 0:45

$$\int f\circ\varphi\cdot\varphi'$$

$$\int \cos x\cdot(\sin^3 x-1)\,dx$$

$\varphi(x)=\sin\varphi$ $\quad$ $\varphi'(x)=\cos x$

$f(x)=x^3-1$

$$\int e^x\cdot(\mathrm{tg}(e^x))\,dx$$

$\varphi(x)=e^x$

$f(x)=\mathrm{tg}\,x$

2)

$1-\cos x$

$$\int\frac{1}{1+\cos x}\,dx=\int\frac{1-\cos x}{1-\cos^2 x}\,dx=\int\frac{1-\cos x}{\sin^2 x}$$

$t=\mathrm{tg}\,\frac{x}{2}$

$$=\int\frac{1}{\sin^2 x}\,dx-\int\frac{\cos x}{\sin^2 x}\,dx$$

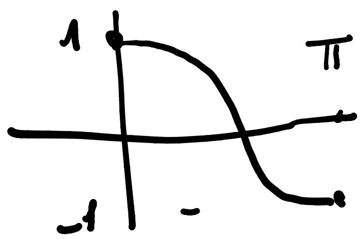
$$\boxed{\begin{matrix}\varphi'(x)=\cos x\\ \varphi(x)=\sin x\\ f(x)=\frac{1}{x^2}\end{matrix}}$$

$\varphi'\cdot f\circ\varphi$

$$\overset{c}{=}-\cot x+\frac{1}{\sin x}$$

$$\int\frac{\cos x}{\sin^2 x}\,dx=\boxed{\left|\begin{matrix}t=\sin x\\ dt=\cos x\,dx\end{matrix}\right|}=\int\frac{1}{t^2}\,dt\overset{c}{=}-\frac{1}{t}$$

$$\int\frac{1}{1+\cos x}\,dx\overset{c}{=}-\left(\cot x-\frac{1}{\sin x}\right)$$



$\frac{1}{1+\cos x}$ $\qquad$ $\mathbb{R}-\{(2k+1)\pi\}$

π

$$\boxed{(-\pi,\pi)+2k\pi}$$

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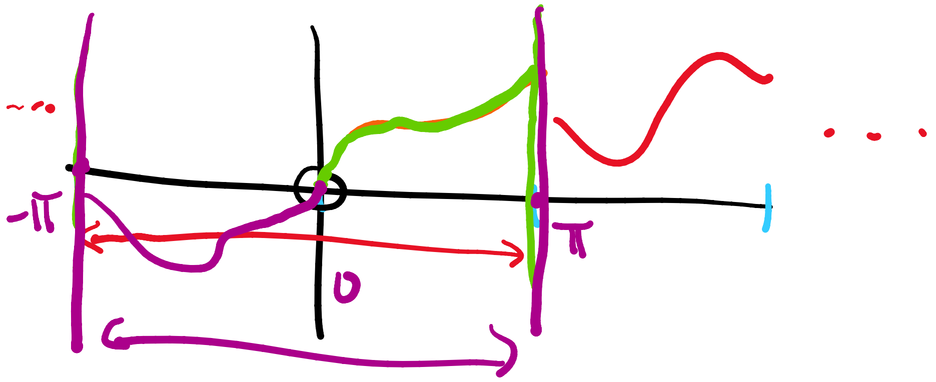
$$-\left(\cot g\, x-\frac{1}{\sin x}\right)=-\left(\frac{\cos x}{\sin x}-\frac{1}{\sin x}\right)$$

$$=-\left(\frac{\cos x-1}{\sin x}\right)\qquad k\pi$$

$$=\frac{1-\cos x}{\sin x}=F(x)\qquad \sin x\neq 0$$

$$\boxed{(0,\pi)+k\pi}$$



$$\lim_{x\to 0^+}F(x)=\lim_{x\to 0^+}\frac{1-\cos x}{\sin x}=\lim_{x\to 0^+}\underbrace{\frac{1-\cos x}{x^2}}_{=\frac{1}{2}}\cdot\underbrace{\frac{x}{\sin x}}_{=1}\cdot\underbrace{x}_{=0}=\underline{\underline{0}}$$

$$\lim_{x\to 0^-}F(x)=\underline{\underline{0}}$$

$$H(x)=\begin{cases}\dfrac{1-\cos x}{\sin x} & x\in\mathbb{R}\setminus\{k\pi\}\\ 0 & x=2k\pi\end{cases}$$

H je PF k $\frac{1}{1+\cos x}$ na $(-\pi,\pi)+2k\pi$ $k\in\mathbb{Z}$.

$t=3x-5$

$$\int \frac{1}{x^2-x+2}\,dx \qquad \frac{1}{a}\int^{a} \sin(3x-5)\,dx = \frac{1}{3}\int \sin t\,dt$$

x

$$\int f(ax+b)\,dx = \left| \begin{array}{l} t = ax+b \\ dt = a\,dx \end{array} \right| = \frac{1}{a}\int f(t)\,dt$$

$$\int \frac{1}{e^x+e^{-x}}\,dx = \int \frac{1}{e^x+e^{-x}} \cdot \boxed{\frac{e^x}{e^x}}\,dx = \int \frac{1}{(e^x)^2+1}\cdot e^x\,dx = \int \frac{1}{t^2+1}\,dt$$

$$e^{-x} = \frac{1}{e^x} \qquad t = e^x \quad dt = e^x\,dx$$

$$\stackrel{c}{=} \operatorname{arctg}(t) = \underline{\underline{\operatorname{arctg}(e^x)}}$$

$$\frac{x-2}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

$$\frac{x-2}{x^1(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$x^3 \to \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x^3}$$

- $$\frac{3x^4+x+1}{(x+1)^3(x+2)^0x^2} = \quad 2 \quad 6 \quad 3 \quad \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

?

$$\frac{x-2}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} = \frac{-1}{x} + \frac{2}{x+2}$$

$x=0$ $\quad$ $x=-2$

$$A = \frac{0-2}{0+2} = -1 \qquad B = \frac{-2-2}{-2} = 2$$

$$\frac{x-2}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$x=0$ $\quad$ $x=-2$

$$A = \frac{0-2}{(0+2)^2} = -\frac{1}{2} \qquad C = \frac{-2-2}{-2} = 2$$

$$\frac{x-2}{x(x+2)^2} - \frac{2}{(x+2)^2} = \frac{-\frac{1}{2}}{x} + \frac{B}{x+2}$$

$$\frac{x-2-2x}{x(x+2)^2} = \frac{-\frac{1}{2}}{x} + \frac{B}{x+2}$$

$$\frac{-x-2}{x(x+2)^2} = \frac{-1}{x\cancel{(x+2)}} = \frac{-\frac{1}{2}}{x} + \frac{B}{x+2}$$

$x=-2$

$$\boxed{B = \frac{-1}{-2} = \frac{1}{2}}$$

$$\boxed{\frac{x-2}{x(x+2)^2} = \frac{-\frac{1}{2}}{x} + \frac{\frac{1}{2}}{x+2} + \frac{2}{(x+2)^2}}$$

$$\int \frac{x-2}{x(x+2)^2}\,dx = -\frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{1}{x+2}\,dx + 2\int \frac{1}{(x+2)^2}\,dx$$

$x=0$

$x=-2$

$$\stackrel{c}{=} -\frac{1}{2}\log|x| + \frac{1}{2}\log|x+2| + 2\,\frac{-1}{x+2}$$

maximální intervaly $(-\infty,-2),(-2,0),(0,\infty)$

$$\frac{2x+1}{x^2(x^2+x+1)^1} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+x+1}$$

$$\frac{2x+1}{x^1(x^2+x+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+x+1} + \frac{Dx+E}{(x^2+x+1)^2}$$

$$\frac{2x+1}{x^2(x^2+x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+x+1}$$

$$2x+1 = Ax(x^2+x+1) + B(x^2+x+1) + (Cx+D)x^2$$

$$0x^3+0x^2+2x+1 = x^3(A+C) + x^2(A+(B+D)) + x(A+B) + B$$

$0 = A+C$ $\quad B=1$

$0 = A+B+D$ $\quad A=1$

$2 = A+B$ $\quad C=-1$

$1 = B$ $\quad D=-2$

$$\int \frac{2x+1}{x^2(x^2+x+1)}\,dx = \int \frac{1}{x}\,dx + \int \frac{1}{x^2}\,dx - \int \frac{x+2}{x^2+x+1}\,dx$$

$$\overset{c}{=} \boxed{\log|x| - \frac{1}{x}}$$

$$\int \frac{x+2}{x^2+x+1}\,dx = \frac{1}{2}\int \frac{2x+4}{x^2+x+1}\,dx = \frac{1}{2}\int \frac{2x+1}{x^2+x+1}\,dx + \frac{3}{2}\int \frac{1}{x^2+x+1}\,dx$$

$'$ $2x+1$

$$\boxed{\int \frac{2x+1}{x^2+x+1}\,dx \overset{c}{=} \log(\underbrace{x^2+x+1}_{>0})}$$

$$\boxed{\int \frac{1}{x^2+x+1}\,dx} = \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}\,dx = \frac{4}{3}\int \frac{1}{\frac{4}{3}\left(x+\frac{1}{2}\right)^2 + 1}$$

$$\frac{4}{3} = \left(\frac{2}{\sqrt{3}}\right)^2 \qquad = \frac{4}{3}\int \frac{1}{\left(\frac{2x+1}{\sqrt{3}}\right)^2 + 1}\,dx$$

1. $x^2+x+1 \quad a = \frac{1}{2}$

$$\boxed{(x+a)^2 + b^2} \qquad b^2 = \frac{3}{4} \qquad x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$x^2 + 2ax + \underbrace{a^2}_{\frac{1}{4}} + b^2 \qquad (x+a)^2 = -b^2$$

$$= \frac{4}{3}\int \frac{1}{\left(\frac{2x+1}{\sqrt{3}}\right)^2 + 1}\,dx = \frac{4}{3}\cdot\frac{\sqrt{3}}{2}\int \frac{1}{t^2+1}\,dt$$

$$\left(\frac{\cdots}{\sqrt{3}}\right)+1$$

$$t=\frac{2x+1}{\sqrt{3}}$$

$$dt=\frac{2}{\sqrt{3}}dx$$

$$\boxed{\stackrel{c}{=}\frac{2}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)}$$

$$\int\frac{2x+1}{x^2(x^2+x+1)}dx\stackrel{c}{=}\boxed{\log|x|-\frac{1}{x}-\frac{1}{2}\log(x^2+x+1)-\frac{3}{2}\frac{2}{\sqrt{3}}\cdot\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)}$$

nie mają wspólne korzeni $\qquad x\in(-\infty,0),(0,\infty)$

$$\int\frac{P(x)}{Q(x)}\qquad \deg Q>\deg P$$

$$\int\frac{x^2+1}{x^2-1}dx=\int\frac{x^2-1+2}{x^2-1}dx=\int 1\,dx+\int\frac{2}{x^2-1}dx$$

$$\int\frac{1}{x(1+2\sqrt{x}+\sqrt[3]{x})}dx=\int\frac{6t^5}{t^6(1+2t^3+t^2)}dt$$

$$t=\sqrt[6]{x}\qquad t^2=\sqrt[3]{x}\qquad t^3=\sqrt{x}$$

$$t^6=x\qquad dx=6t^5dt$$

$$t=\left(\frac{ax+b}{cx+d}\right)^{\frac{1}{m}}$$

$$\int\frac{1}{x+\sqrt{x+1}}dx\qquad \int\frac{1+\sqrt{\frac{x+1}{x-1}}}{1+x^2}dx$$

$t=\sqrt{x+1}$ $\qquad$ $t=\sqrt{\frac{x+1}{x-1}}$

- $\displaystyle\int \frac{\sin^2 x}{1+\sin^2 x}\,dx =$

- $\displaystyle\int \frac{1}{\sin^2 x-2\cos^4 x}\,dx$

- $\displaystyle\int \frac{\sin x\cos x+1}{\sin^2 x+4}$

$\sin^2 x=\frac{t^2}{1+t^2}$ $\qquad$ $\cos^2 x=\frac{1}{1+t^2}$ $\qquad$ $\sin x\cos x=\frac{t}{1+t^2}$

$t=\operatorname{tg} x$ $\qquad$ $dx=\frac{1}{1+t^2}\,dt$

$x=\operatorname{arctg} t$

$$\int\frac{\sin^2 x}{1+\sin^2 x}\,dx=\int\frac{\frac{t^2}{1+t^2}}{1+\frac{t^2}{1+t^2}}\cdot\frac{1}{1+t^2}\,dt$$

$$=\int\frac{t^2}{(1+t^2)(1+2t^2)}\,dt$$

- $\displaystyle\int \frac{1}{2\sin x-\cos x+5}\,dx$

$$t=\operatorname{tg}\frac{x}{2} \qquad \cos x=\frac{1-t^2}{1+t^2} \qquad \sin x=\frac{2t}{1+t^2}$$

$$x=2\operatorname{arctg} t \qquad dx=\frac{2}{1+t^2}$$

$$\int \frac{1}{\frac{4t}{1+t^2}-\frac{1-t^2}{1+t^2}+5}\cdot\frac{2}{1+t^2}\,dt=\int\frac{2}{6t^2+4t+4}\,dt$$

$$\frac{4t-1+t^2+5t^2+5}{1+t^2}(1+t^2) \qquad =\int\frac{1}{3t^2+2t+2}\,dt$$

$$\int\frac{1}{3t^2+2t+2}\,dt=\frac{1}{3}\int\frac{1}{t^2+\frac{2}{3}t+\frac{2}{3}}=\frac{1}{3}\int\frac{1}{\left(t+\frac{1}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}\,dt$$

$$t^2+\frac{2}{3}t+\frac{2}{3}=(t+a)^2+b^2=\left(t+\frac{1}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2$$

$$2at$$

$$2a=\frac{2}{3} \quad a=\frac{1}{3} \qquad a^2=\frac{1}{9} \qquad a^2+b^2=\frac{2}{3}$$

$$b^2=\frac{2}{3}-\frac{1}{9}=\frac{6}{9}-\frac{1}{9}=\frac{5}{9}$$

$$=\frac{1}{3}\int\frac{1}{\left(t+\frac{1}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}\,dt=\frac{1}{3}\cdot\frac{9}{5}\int\frac{1}{\left[\frac{3}{\sqrt{5}}\left(t+\frac{1}{3}\right)\right]^2+1}\,dt$$

$$a=\frac{3}{\sqrt{5}} \qquad \frac{3t+1}{\sqrt{5}}=u \qquad \frac{3}{\sqrt{5}}t+\frac{1}{\sqrt{5}}$$

$$=\frac{\sqrt{5}}{3}\cdot\frac{1}{3}\cdot\frac{9}{5}\int\frac{1}{u^2+1}\,du \overset{S}{=} \frac{1}{\sqrt{5}}\operatorname{arctg}\left(\frac{3\operatorname{tg}\frac{x}{2}+1}{\sqrt{5}}\right)$$

$$(-\pi, \pi) + 2k\pi$$