úterý 22. prosince 2020 13:37

$$\lim_{n\to\infty} \frac{311n}{3^n} = 0 \rightarrow 311n \le C\cdot 3^n$$

$$T^n_{a,f}(x) = f(a) + \sum_{k=1}^{n} \frac{f^{(k)}(a)}{k!}\cdot \underline{(x-a)^k}$$

$$\uparrow$$

$$\underline{(x-1)^k}$$

(1) $f(x) = \sqrt{x}$ $a=1$ $n=3$

$$1 = f(a),\ 1 = \frac{f'(a)}{1!},\ \frac{f''(a)}{2!},\ \frac{f'''(a)}{3!}$$

$$f(a) = \sqrt{1} = 1$$

$$(\sqrt{x})' = (x^{\frac12})' = \frac12 \cdot \frac{1}{x^{\frac12}} \xrightarrow{x=1} \frac12$$

$$\left(\frac12 x^{-\frac12}\right)' = -\frac14 x^{-\frac32} \xrightarrow{x=1} -\frac14 \cdot \frac12 = -\frac18$$

$$\left(-\frac14 x^{-\frac32}\right)' = \boxed{\frac38} x^{-\frac52} \xrightarrow{x=1} \boxed{\frac38}\cdot\frac16 = \frac{1}{16}$$

$$T^3_{1,\sqrt{\ }}(x) = 1 + \frac12(x-1) - \frac18(x-1)^2 + \frac{1}{16}(x-1)^3$$

$\sin^2 x$ $\underline{n=5}$ $a=0$

$\sin x$ $\qquad\qquad n=5 \quad n=0$

$\sin x \cdot \sin x$

$$T^5_{0,\sin x}(x) = x - \frac{x^3}{6} + \frac{x^5}{120} = T^6_{0,\sin x}$$

$$T^5_{0,\sin^2 x}(x) = T^5_{0,\sin x}(x) \cdot T^5_{0,\sin x}(x) + o(x^5)$$

$$= \left(x - \frac{x^3}{6} + \frac{x^5}{120}\right)\left(x - \frac{x^3}{6} + \frac{x^5}{120}\right) + o(x^5)$$

$$= x^2 + x^4\left(-\frac{1}{6} - \frac{1}{6}\right) = x^2 - \frac{x^4}{3} + o(x^5)$$

$$\underline{T^6_{0,\sin^2 x}(x)} = T^6_{0,\sin x} \cdot T^6_{0,\sin x} + o(x^6)$$

$$= \left(x - \frac{x^3}{6} + \frac{x^5}{120}\right)\left(x - \frac{x^3}{6} + \frac{x^5}{120}\right) + o(x^6)$$

$$= x - \frac{x^4}{3} + x^6\left(\frac{1}{120} + \frac{1}{120} + \frac{1}{36}\right) + o(x^6)$$

$$\frac{1}{60} + \frac{1}{36} = \frac{3}{180} + \frac{5}{180} = \frac{8}{180} = \underline{\underline{\frac{2}{45}}}$$

$$\boxed{T^6_{0,\sin^2 x}(x) = x - \frac{x^4}{6} + \frac{2}{45}x^6}$$

$$T^6_{0,\sin(\sin x)}$$

$\sin \underline{\underline{0}} = \underline{\underline{0}}$

(3) (5)

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)$$

$$\underline{\sin(\sin x)} = \left(x - \frac{x^3}{6} + \frac{x^5}{120}\right) - \frac{1}{6}\left(x - \frac{x^3}{6} + \frac{x^5}{120}\right)^3$$

$(\cdot)(\cdot)(\cdot)$

$(\cdot)(\cdot)(\cdot)$

$$+ \frac{1}{120}\left(x - \frac{x^3}{6} + \frac{x^5}{120}\right)^5 + o(x^5)$$

$o(x^5)$

$$= x + x^3\left(-\frac{1}{6} - \frac{1}{6}\right) + x^5\left(\frac{1}{120} + 3\cdot\left(-\frac{1}{6}\right)\left(-\frac{1}{6}\right) + \frac{1}{120}\right)$$

$$= \boxed{x - \frac{x^3}{3} + \frac{1}{10}x^5} + o(x^5)$$

$$\frac{1}{60} + \frac{1}{12} = \frac{6}{60} = \frac{1}{10}$$

(7)

$$\lim_{x \to 0} \frac{\cos x - e^{-\frac{x^2}{2}}}{x^4}$$

$$T^4_{0, e^x}(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$$

$T^4_{0, \cos x - e^{-\frac{x^2}{2}}}$

$\left(e^{-\frac{x^2}{2}}\right.$ je sudá$\left.\right)$

$$T^4_{0, e^{-\frac{x^2}{2}}}(x) = 1 + 0x - \frac{x^2}{2} + 0x^3 + \frac{x^4}{8}$$

$$T^4_{0, \cos x}(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

$T^4_{0,\cos x}$ ... 24

$n \geq 0 \quad T^n_{0,-\frac{x^2}{2}}(x) = -\frac{x^2}{2}$

$T^0_{0,-\frac{x^2}{2}}(x) = T^1_{0,-\frac{x^2}{2}}(x) = 0$

$T^4_{0,e^{-\frac{x^2}{2}}} = 1 - \frac{x^2}{2} + \frac{x^4}{8}$

$T^4_{0,\cos x} = 1 - \frac{x^2}{2} + \frac{x^4}{24}$

$T^4_{0,\cos x - e^{-\frac{x^2}{2}}} = \frac{x^4}{24} - \frac{x^4}{8} = -\frac{x^4}{12}$

$\varphi(x) \cdot x^4 \quad$ "$o(x^4)$" $\quad \frac{o(x^4)}{x^4} \to 0$ $\quad \varphi(x) \to 0 \quad x \to 0$

$$\lim_{x\to 0} \frac{\cos x - e^{-\frac{x^2}{2}}}{x^4} = \lim_{x\to 0} \frac{-\frac{x^4}{12} + o(x^4)}{x^4} = -\frac{1}{12}$$

$$\lim_{x\to 0} \frac{\overbrace{e^x \sin x - x(x+1)}^{f(x)}}{x^3}$$

$T^3_{0,f}(x) =$

$x(x+1) = x^2 + x$

$e^x \sin x = \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)\left(x - \frac{x^3}{6}\right) + o(x^3)$

$$e^{x}\sin x = \left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)\left(x-\frac{x^3}{6}\right) + o(x^3)$$

$$= x + x^2 + x^3\left(-\frac{1}{6}+\frac{1}{2}\right) = x + x^2 + \frac{x^3}{3}$$

$$\lim_{x\to 0} \frac{e^{x}\sin x - x - x^2}{x^3} = \lim_{x\to 0} \frac{\frac{x^3}{3} + o(x^3)}{x^3} = \frac{1}{3}$$

$$= \frac{1}{3} + \lim_{x\to 0} \underbrace{\frac{o(x^3)}{x^3}}_{=0}$$

12) $\lim\limits_{x\to 0} \dfrac{\cos x - \cos(\sin x)}{x^4} \neq 0, +\infty$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} \;\dots$$

$$\cos(\sin x) = 1 - \frac{1}{2}\left(x - \frac{x^3}{6}\right)^2 + \frac{1}{24}\left(x - \frac{x^3}{6}\right)^4$$

$$= 1 - \frac{x^2}{2} + x^4\left(2\cdot\left(-\frac{1}{6}\right)\cdot\left(-\frac{1}{2}\right) + \frac{1}{24}\right)$$

$$= 1 - \frac{x^2}{2} + x^4 \cdot \frac{5}{24} + o(x^4)$$

$$\lim_{x\to 0} \frac{\cos x - \cos(\sin x)}{x^4} = \lim_{x\to 0} \frac{\frac{1}{24}x^4 - \frac{5}{24}x^4 + o(x^4)}{x^4}$$

$$= -\frac{1}{6}$$

$\underline{6}$

e s chybou $\underline{10^{-4}}$

$$e = e^1 = \underbrace{1 + 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \dots + \frac{1}{n!}} + R$$

$$f(x) - T^n_{0,f}(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}.$$

$f(x) = e^x$

$$|R^n_{0,e^x}| \leq \left| \frac{e^\xi}{(n+1)!} x^{n+1} \right| \qquad x = 1$$

(a, x)

$$= \frac{e^\xi}{(n+1)!} \cdot 1 \leq \frac{e}{(n+1)!} \qquad e^\xi \qquad \xi \in (0,1)$$

$e \leq 4$

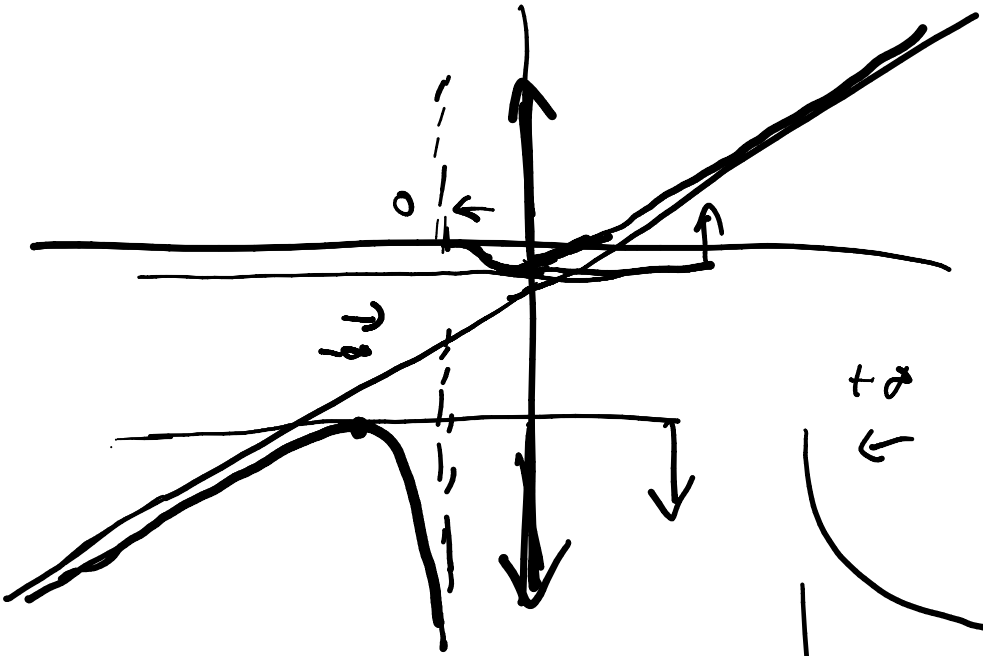
$$|e - T^n_{0,e^x}(1)| \leq \frac{e}{(n+1)!} \leq \frac{4}{(n+1)!}$$

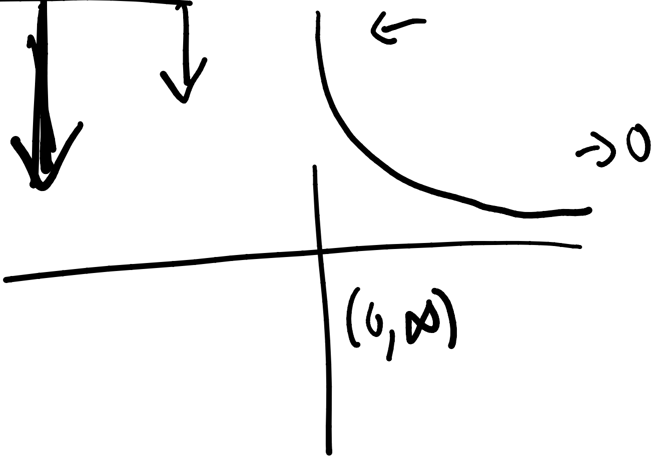
pro jaké $n \in \mathbb{N}$ $\quad \frac{4}{(n+1)!} \leq \frac{1}{10000}$

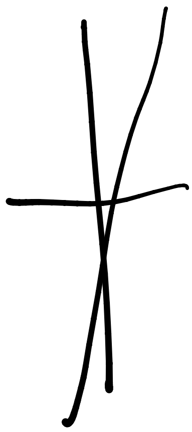
$(1 + \frac{1}{n})^n \to \dots$ $\quad \boxed{(n+1)! \geq 40000}$

$(x-1)e^{\frac{x}{x+1}}$

$ex-2e$







$(0,\infty)$

$\mathbb{R} \cup \{\pm\infty\}$